



# Magma degassing during the Plinian eruption of Novarupta, Alaska, 1912

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[1] We have modeled the nucleation and isothermal growth of bubbles in dacite from the 1912 Plinian eruption of Novarupta, Alaska. Bubble growth calculations account for the exsolution of H<sub>2</sub>O and CO<sub>2</sub>, beginning with bubble nucleation and ending when bubble sizes reproduced the observed size distribution of vesicles in Novarupta pumice clasts. Assuming classical nucleation theory, bubbles nucleated with a diameter of the order of 10<sup>-8</sup> m and grew to sizes ranging from 10<sup>-6</sup> m to greater than 10<sup>-3</sup> m, the typical range of vesicle sizes found in Novarupta pumice. The smallest vesicles in Novarupta pumices are also the most abundant and bubbles with radii of 10<sup>-6</sup> m to 10<sup>-5</sup> m comprise almost 90% of the entire bubble population. We find that these bubbles must have nucleated and grown to their final size within a few 100 milliseconds. Despite these extremely fast growth rates, the pressures of exsolved volatiles contained within the bubbles remained high, up to about 10<sup>7</sup> Pa in excess of ambient pressure. Assuming a closed-system, the potential energy of these compressed volatiles was sufficient to cause magma fragmentation, even though only a fraction of the pre-eruptive volatiles had exsolved. Unless the matrix glasses of Novarupta pyroclasts retains a large fraction of pre-eruptive volatiles, the majority of magmatic volatiles (80–90%) was likely lost by open-system degassing between magma fragmentation and quenching.

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## 1. Introduction

[2] Styles of volcanic eruptions range from effusive to highly explosive. Because of their tremendous destructiveness, the latter pose a considerable threat to human life, economies and the environment. Because

magma ascent is inaccessible to direct observation, eruptive products, pyroclasts in the case of explosive eruptions, provide the best direct record of eruptive conditions. In particular bubbles, which are preserved as vesicles within these pyroclasts, have been intensely studied with the objective of constraining



the conditions of magma ascent [e.g., *Sparks and Brazier*, 1982; *Mangan et al.*, 1993; *Klug and Cashman*, 1994; *Gardner et al.*, 1996; *Klug and Cashman*, 1996; *Mangan and Cashman*, 1996; *Hammer et al.*, 1999; *Blower et al.*, 2001a; *Polacci et al.*, 2001; *Blower et al.*, 2002; *Klug et al.*, 2002; *Polacci et al.*, 2003; *Gaonac'h et al.*, 2003, 2005; *Polacci*, 2005; *Polacci et al.*, 2006a, 2009; *Gurioli et al.*, 2005; *Polacci et al.*, 2006b; *Adams et al.*, 2006; *Lautze and Houghton*, 2007; *Piochi et al.*, 2008; *Giachetti et al.*, 2010; *Houghton et al.*, 2010; *Shea et al.*, 2010; *Giachetti et al.*, 2011; *Voltolini et al.*, 2011].

[3] Our study is focused on the eruption dynamics of Earth's most voluminous volcanic eruption of the 20th century, the Plinian eruption of Novarupta on June 6–8, 1912 [*Fierstein and Hildreth*, 1992; *Hildreth and Fierstein*, 2000]. We have constrained magma ascent conditions during Episode III of this eruption, by reconstructing the vesicle size distributions in pyroclasts from this episode, using accurate modeling of diffusive bubble growth during the exsolution of a mixed H<sub>2</sub>O and CO<sub>2</sub> magmatic volatile phase [*Gonnermann and Manga*, 2005]. The 1912 eruption of Novarupta is widely recognized as an outstanding example of sustained Plinian explosive volcanism. It has been the subject of a century of intense study, resulting in a well-developed tephrostratigraphy [*Hildreth and Fierstein*, 2012, and references therein] and extensive textural analysis of air-fall deposits [*Adams et al.*, 2006], as well as thorough characterization of pre-eruptive magma storage conditions [*Hammer et al.*, 2002].

[4] The cause of explosive volcanism is the sudden release and rapid expansion of a volatile (gas) phase. During magma ascent the solubility of magmatic volatiles, to a large extent H<sub>2</sub>O and CO<sub>2</sub>, decreases as the confining pressure on the magma decreases. Consequently, the melt becomes supersaturated in volatiles, which results in the nucleation of bubbles by the exsolution of magmatic volatiles (gases). These bubbles will grow, due to diffusion of volatiles from the melt and the expansion of the already exsolved volatiles. During many explosive eruptions bubbles cannot grow as fast as pressure decreases in the surrounding melt by which they are transported to the surface. This is a consequence of the strong dependence of melt viscosity on the concentration of dissolved H<sub>2</sub>O, which causes melt viscosity to increase upon H<sub>2</sub>O exsolution. If the viscous time-scale, which is approximately equal to the ratio of melt viscosity to the excess pressure inside bubbles,

is larger than the time over which magma decompresses, then bubbles will not be able to grow fast enough to expand and equilibrate with the ambient pressure [*Lensky et al.*, 2004]. Consequently, the bubbles may be of considerably higher pressure than the surrounding melt.

[5] If this bubble overpressure exceeds a certain threshold [*McBirney and Murase*, 1970; *Alidibirov*, 1994; *Zhang*, 1999; *Spieler et al.*, 2004a; *Mueller et al.*, 2008], or if it causes sufficiently high rates of bubble growth [*Dingwell*, 1996; *Papale*, 1999], the viscoelastic rheology of the melt will cause it to fragment in a brittle manner. Fragmentation allows the pressurized volatiles to escape from their confinement and expand explosively in volume. This abrupt transition, from a continuous melt phase with suspended bubbles, to an explosively expanding gaseous flow with suspended magma fragments, called pyroclasts, is sustained for hours during Plinian eruptions. Pyroclasts are transported from their depth of fragmentation into the Earth's atmosphere, at velocities of hundreds of meters per second, by the rapidly expanding gases. When the pyroclasts settle out of this Plinian eruption column, they form aerially extensive pyroclastic air-fall deposits, with potentially severe environmental and societal impacts. Consequently, an understanding of the magmatic conditions leading up to and after magma fragmentation is of key importance in the study of explosive volcanism.

[6] A record of these conditions are the size distribution and number density of vesicles in the erupted pyroclasts. Number density is defined as number of vesicles per unit volume of solid matrix, presumably equivalent to the number of bubbles per unit volume of melt, prior to quenching. A question of fundamental importance is whether the ascending magma traverses a region of extreme decompression rates, of the order of 100 MPa s<sup>-1</sup> [e.g., *Toramaru*, 1995, 1989, 2006]. It is unclear from experiments if ~100 MPa s<sup>-1</sup> is a necessary requirement to achieve the high bubble number densities observed in pyroclasts [*Mourtada-Bonnefoi and Laporte*, 2004; *Hamada et al.*, 2010; *Gardner and Ketcham*, 2011; *Gonde et al.*, 2011; *Nowak et al.*, 2011]. Relatively high rates of decompression are required to outpace the diffusion of dissolved volatiles into existing bubbles [e.g., *Sparks*, 1978; *Proussevitch et al.*, 1993; *Lyakhovskiy et al.*, 1996; *Gardner et al.*, 1999; *Mourtada-Bonnefoi and Laporte*, 2002, 2004; *Lensky et al.*, 2004; *Gonnermann and Manga*, 2007], thereby driving the melt toward progressively increasing supersaturation and increasing rates of bubble nucleation. If at all, decompression



rates of  $\sim 100 \text{ MPa s}^{-1}$  may only be plausible for a very short duration and within a short distance of the fragmentation depth, where viscosity may increase considerably over short distances, due to the exsolution of  $\text{H}_2\text{O}$  [e.g., Woods, 1995; Papale, 1999; Mastin, 2002; Koyaguchi, 2005; Massol and Koyaguchi, 2005; Melnik et al., 2005]. For a given flow rate, and assuming Newtonian rheology, viscous resistance within the conduit requires that pressure decreases concurrently. This so-called frictional pressure loss can result in steep pressure gradients and high rates of decompression. If the latter outpaces the decrease in pressure within the expanding bubbles, the resulting difference in pressure between the interior of bubbles and the surrounding melt may exceed the overpressure at which magma will fragment [Alidibirov, 1994; Dingwell, 1996; Papale, 1999; Zhang, 1999; Spieler et al., 2004a]. However, the buildup of overpressure may be modulated or balanced by the development of interconnected bubbles, amenable to gas loss from the erupting magma by permeable gas flow [Burgisser and Gardner, 2005; Mueller et al., 2008; Rust and Cashman, 2011]. The objective of our study is to constrain the rates of magma decompression required to produce the vesicle size distributions in fall deposits from Episode III of the 1912 Novarupta eruption and to examine the relationship between bubble nucleation, growth and fragmentation.

[7] Section 2 gives a brief summary of the 1912 Novarupta eruption, its deposits and the samples modeled herein. In section 3 we provide a review of the quantitative description of vesicles in pyroclasts. It is followed by a discussion of bubble nucleation in section 4, the modeling thereof, as well as a brief synopsis of the role of surface tension in studies of bubble nucleation and the associated uncertainties. Section 5 presents the conceptual framework for a new approach to the modeling of vesicle size distributions, based on the modeling of diffusive bubble growth, described in section 6, followed by a step-by-step description of the model in section 7. Finally, section 8 presents a discussion of the model results, followed by conclusions in section 9.

## 2. Novarupta 1912

### 2.1. Eruption and Deposits

[8] The 1912 Novarupta eruption was the most voluminous eruption of the twentieth century. Because caldera collapse occurred 10 km from the vent, eruption deposits have been well preserved

close to the vent. The explosive phase of the eruption began on 6 June, 1912 and lasted for 60 hours, with Plinian fall deposits and voluminous ignimbrites deposited during parts of the eruption [e.g., Hildreth, 1983; Fierstein and Hildreth, 1992; Hildreth and Fierstein, 2000, 2012]. Based on tephrostratigraphic relations and eyewitness accounts, it appears that there were two short breaks in the Plinian phase of the eruption, one on the morning of 7 June and the other in the evening of 8 June. Consequently, the explosive phase of the eruption is subdivided into three episodes, with inferred column heights of 26, 25, and 23 km respectively. Most of the ignimbrite was deposited during Episode I, whereas Episodes II and III predominantly formed Plinian fall deposits. After the explosive phase, two episodes of dome emplacement are designated as Episodes IV and V.

### 2.2. Pre-eruptive Volatile Content and Pressure

[9] The erupted magmas are rhyolite (77–78 wt%  $\text{SiO}_2$ ), dacite (63–68 wt%  $\text{SiO}_2$ ) and andesite (58–63 wt%  $\text{SiO}_2$ ) [Hildreth and Fierstein, 2000]. Melt inclusions in quartz phenocrysts of the Novarupta rhyolite contain dissolved  $\text{H}_2\text{O}$  concentrations between 3.5 and 4.7 wt% and melt inclusions in plagioclase phenocrysts in dacite contain between 2.2 and 3.1 wt%  $\text{H}_2\text{O}$  [Lowenstern, 1993].

[10] The  $\text{CO}_2$  content of both rhyolites and dacites is less than 50 ppm [Lowenstern, 1993; Wallace, 2005]. We focus our study on the 1912 Novarupta dacite from Episode III. Using experimental phase relations at  $\text{H}_2\text{O} + \text{CO}_2$  fluid saturation, Hammer et al. [2002] determined a  $\text{H}_2\text{O}$  saturation pressure of approximately 50 MPa at  $850^\circ\text{C}$  at a total pressure of 50–150 MPa for the dacite. However, concentrations of less than 50 ppm  $\text{CO}_2$  in melt inclusions, suggests relatively limited  $\text{H}_2\text{O}$  undersaturation, caused by the presence of a  $\text{CO}_2$ -rich vapor phase [Blundy et al., 2010]. An alternate possibility suggested by Hammer et al. [2002] is  $\text{H}_2\text{O}$  undersaturation, due to a lack of  $\text{H}_2\text{O}$ .

[11] The presence of  $\text{CO}_2$  can modulate the nucleation rate of bubbles [Mourtada-Bonnefoi and Laporte, 2002; Yamada et al., 2005; Larsen, 2008] and affect eruption dynamics [Papale and Polacci, 1999], especially at shallow depths [Burgisser et al., 2008]. In our model we, therefore, focus on the dacite end-member suggested by Hammer et al. [2002], which equilibrated at  $850^\circ\text{C}$ , a  $\text{H}_2\text{O}$  saturation pressure of 50 MPa, and a vapor phase with 10 mole % of  $\text{CO}_2$ .



### 2.3. Existing Textural Analyses of Novarupta Plinian Fall Pumices

[12] *Adams et al.* [2006] carried out a detailed textural study of the 1912 Novarupta Plinian fall deposits. They sampled and texturally characterized dacite pumice clasts from Episodes II and III and the modeling described herein builds upon this analysis. *Adams et al.* [2006] obtained vesicle size distributions through a two-dimensional analysis of Scanning Electron Microscope (SEM) images from polished thin sections for a subset of pumice clasts of individual sample suites. Number densities of vesicles were calculated for binned vesicle diameters thus obtained and converted from two-dimensional to three-dimensional number densities using the method of *Sahagian and Proussevitch* [1998]. This methodology may result in some bias, due to the inherent spherical assumption. However, given the present limitation associated with the estimation of VSDs from three-dimensional Computed Tomography (CT) scans, the method employed by *Adams et al.* [2006] provides the most robust estimate of VSDs for the Novarupta samples [*Giachetti et al.*, 2011].

### 2.4. Qualitative Interpretation of Novarupta Plinian Fall Pumices

[13] Similarities in bubble number density,  $N_m$ , and vesicle size distributions,  $n(a)$ , of Novarupta pyroclasts have been interpreted to signify similar magma ascent conditions [*Adams et al.*, 2006]. Near-unimodal vesicle volume fractions, with a relatively invariant mode throughout episodes II and III, are thought to indicate that magma ascent conditions remained relatively constant, with bubble nucleation and growth occurring over a short time interval [*Toramaru*, 1989, 1990; *Lyakhovskiy et al.*, 1996; *Gardner et al.*, 1999; *Adams et al.*, 2006]. At the same time, the abundance of small vesicles are thought to indicate continuous nucleation until the late stages of magma ascent [*Adams et al.*, 2006], presumably associated with homogenous nucleation at large supersaturations [*Hurwitz and Navon*, 1994; *Gardner et al.*, 1999; *Mourtada-Bonnefoi and Laporte*, 1999; *Mangan and Sisson*, 2000]. Because the distribution  $n(a)$  is considerably more skewed toward small vesicles than would be expected for steady state nucleation and growth [*Lyakhovskiy et al.*, 1996; *Blower et al.*, 2001b; *Klug et al.*, 2002; *Adams et al.*, 2006], the bubble nucleation rate may have increased with time, perhaps modulated by bubble coalescence [*Gaonac'h et al.*, 1996; *Simakin et al.*, 1999; *Mangan et al.*, 2004].

### 2.5. Modeled Samples

[14] We model three Plinian dacite fall clasts from Episode III, Unit G, sampling site 94-Z, sample No. 22 [*Adams et al.*, 2006]. They are named 94-Z-22-28, 94-Z-22-24 and 94-Z-22-8 and represent high, modal and low-density clasts with a vesicularity of 75.2%, 63.5% and 51.7%, respectively [see *Adams et al.*, 2006, Figure 8]. Bubble number density ranges between  $\sim 10^{14}$  and  $\sim 10^{15} \text{ m}^{-3}$ , with median vesicle diameter between 45 and 74  $\mu\text{m}$  for vesicles ranging in size between 4 and 1989  $\mu\text{m}$  (see Table 1 of *Adams et al.* [2006] for details). Various stages of bubble coalescence are evident in the samples and the walls of coalesced bubbles are approximately 1–3  $\mu\text{m}$  in thickness, similar to Plinian dacites and rhyolites elsewhere [*Klug and Cashman*, 1996; *Klug et al.*, 2002].

[15] Novarupta dacites contain phenocrysts (predominantly plagioclase with minor clinopyroxene and magnetite) [*Hildreth*, 1983] and a few plagioclase microlites [*Adams et al.*, 2006]. Because plagioclase is not amenable to nucleation and because the size and number density of bubbles is not significantly higher around phenocrysts, heterogenous bubble nucleation associated with crystals is not likely to have been of importance at Novarupta [e.g., *Hurwitz and Navon*, 1994; *Gardner and Denis*, 2004; *Mourtada-Bonnefoi and Laporte*, 2004; *Gardner*, 2007a; *Larsen*, 2008].

### 3. Vesicle Size Distributions in Pyroclasts

[16] Vesicle size distributions (VSDs) in pyroclasts are frequently used to infer conditions of syn-eruptive magma vesiculation, that is bubble nucleation, growth and coalescence. VSDs represent one example of what is known generically as a population distribution. The standard approach to quantitative studies of population distributions is the population balance equation (PBE), which describes the temporal change in a population distribution due to the addition (*birth*), change in size (*growth*) and loss (*death*) of individual members of the distribution. In our case, *birth* denotes bubble nucleation, as a consequence of volatile supersaturation of the melt. *Growth* refers to bubble growth as a consequence of decreasing pressure and exsolution of volatiles. It could also include a decrease in bubble size, due to open-system gas loss, which is not considered explicitly herein. Last, *death* refers to bubble coalescence.



[17] Consequently, a quantitative interpretation of VSDs requires the estimation of nucleation rate  $J(t)$ , growth rate  $G(a, t)$  and coalescence rate  $H(a, t)$ , either by forward or by inverse modeling. However, one of the challenges lies in the difficulty of solving the population balance equation [Ramkrishna, 2000]. Much of this difficulty arises due to the coalescence term, which, despite recent progress [e.g., Larsen and Gardner, 2000; Burgisser and Gardner, 2005; Gardner, 2007b; Bai et al., 2008; Gonde et al., 2011], remains to a large extent insufficiently constrained as an a-priori model parameter [e.g., Gaonac'h et al., 1996; Klug and Cashman, 1996; Herd and Pinkerton, 1997; Blower et al., 2002; Gaonac'h et al., 2003, 2005; Lovejoy et al., 2004]. We therefore do not model bubble coalescence. After providing a brief review on the quantitative descriptions of VSDs in this section, we discuss bubble nucleation and growth in subsequent sections.

### 3.1. Number Density

[18] The total number of vesicles in a sample is given by

$$n_{\text{tot}} = \sum n_i, \quad (1)$$

where  $n_i$  is the number of vesicles per size class  $i$ , which frequently is defined in terms of vesicle radius or volume. If, for example,  $a$  denotes vesicle radius, then there are  $n_i$  vesicles of radius  $a_i < a \leq (a_{i+1})$ . Because of the wide range in vesicle sizes found in pyroclasts, the value of  $a_i$  is often taken to increase geometrically by a factor of  $\delta$ , so that  $\log a_i = i\delta$ . VSDs in Novarupta pyroclasts have been characterized with  $\delta = 0.1$  [e.g., Adams et al., 2006].

[19] The total number of vesicles per unit volume of matrix, called the vesicle number density, is given by

$$N_m = \frac{n_{\text{tot}}}{v_m}, \quad (2)$$

where  $v_m$  is the volume of matrix, which usually is comprised of glass or glass plus microlites. Here we define  $N_m$  as the number of vesicles per unit volume of glass matrix. Although bubbles may continue growing during the time interval between magma fragmentation and pyroclast quenching [Thomas et al., 1994; Gardner et al., 1996], especially in large clasts or where cooling rate is slow, conventionally it is thought that  $N_m$  is a reasonable approximation to the bubble number density (BND), the number of bubbles per unit volume melt prior to quenching of the pyroclast [e.g., Toramaru, 2006]. Therefore, we shall use the terms vesicle number

density and bubble number density interchangeably herein.  $N_m$  provides a time-integrated record of bubble nucleation, albeit potentially modulated by bubble coalescence. It is found the  $N_m$  varies considerably among pyroclasts from various eruptions [Toramaru, 2006; Rust and Cashman, 2011].

[20] The vesicle number density,  $n(a_i)$ , and the exceedance density,  $N(a_i)$ , are two frequently used representations of VSDs. They are defined, respectively, as the number of vesicles within a given size interval per volume of solid matrix

$$n(a_i) = \frac{1}{v_m} \frac{n_i}{(a_{i+1} - a_i)}, \quad (3)$$

the number of vesicles per volume of matrix that are greater than or equal to size  $a_i$

$$N(a_i) = \sum_i^{\infty} n(a_i). \quad (4)$$

### 3.2. Volume Fraction Density

[21] Vesicle size distributions in pumice are frequently logarithmic [Proussevitch et al., 2007] and detailed population characteristics, such as multiple modes, may not be easily recognizable on logarithmically plotted distribution densities  $n(a_i)$  or  $N(a_i)$ . Vesicle volume fractions,  $\phi(a_i)$ , also called vesicle volume distributions (VVDs), are not logarithmically distributed, because of the decrease in volume for small vesicle sizes.  $\phi(a_i)$  is defined as the volume of vesicles within a given size range, divided by the total volume of vesicles in the sample,

$$\phi(a_i) = \frac{\sum_{a=a_i}^{a_{i+1}} n(a)a^3}{\sum_{a=0}^{\infty} n(a)a^3}, \quad (5)$$

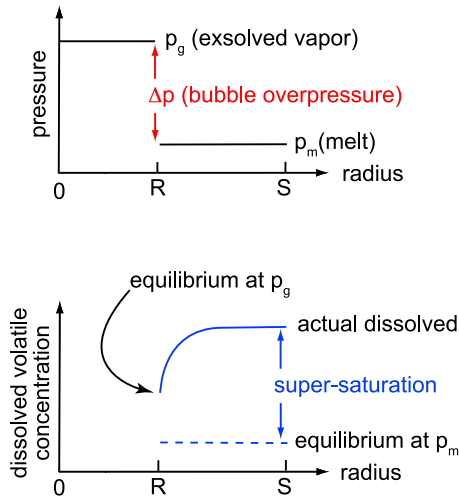
and provides a useful complement to  $n(a_i)$  and  $N(a_i)$  [Sparks and Brazier, 1982; Klug and Cashman, 1994; Mangan and Cashman, 1996; Herd and Pinkerton, 1997; Klug et al., 2002; Shea et al., 2010].

## 4. Bubble Nucleation

### 4.1. Classical Nucleation Theory

[22] Based on classical nucleation theory [Hirth et al., 1970; Sparks, 1978; Hurwitz and Navon, 1994; Mangan and Sisson, 2005], the rate of bubble nucleation,  $J$ , is thought to depend on the volatile supersaturation of the melt as

$$J = C \exp\left(-\frac{16\pi\sigma^3}{3k_B T \Delta P_s^2} \psi\right). \quad (6)$$



**Figure 1.** Schematic diagram illustrating supersaturation pressure,  $\Delta P_s$ , the difference between the pressure at which the melt would be saturated (in equilibrium) with the dissolved volatiles,  $p_{\text{sat}}$ , and the actual pressure,  $p_m$ .

Here  $\sigma$  is the surface tension between the melt and the exsolved magmatic volatiles,  $k_B = 1.3805 \times 10^{-23}$  J/K is the Boltzmann constant and  $T$  is the absolute temperature.  $0 \leq \psi \leq 1$  is a geometrical factor that accounts for the presence of impurities (e.g., crystals), which lower the interfacial energy associated with the formation of a stable bubble nucleus [e.g., Hurwitz and Navon, 1994; Gardner and Denis, 2004; Mourtada-Bonnefoi and Laporte, 2004; Gardner, 2007a; Larsen, 2008].  $C$  is a pre-exponential factor and  $\Delta P_s$  is the supersaturation pressure given by

$$\Delta P_s = p_{\text{sat}} - p_m, \quad (7)$$

where  $p_m$  is the pressure of the melt and  $p_{\text{sat}}$  is the pressure at which melt would be saturated with the actual concentration of dissolved volatiles, assuming equilibrium conditions (Figure 1). For silicate melts,  $C$  is usually given as [Hirth et al., 1970; Sparks, 1978; Hurwitz and Navon, 1994; Mangan and Sisson, 2005]

$$C = \frac{2n_0^2 V_m D}{a_0} \sqrt{\frac{\sigma}{k_B T}}. \quad (8)$$

Here  $n_0$  is the number of dissolved volatile molecules per volume of liquid (melt),  $V_m$  is the volume of a volatile molecule,  $D$  is the diffusivity of the volatile species in the liquid (melt) and  $a_0 \approx n_0^{-1/3}$  is the mean distance between dissolved volatile molecules. Note that  $n_0 = (N_A X \rho_m / M)$ , where  $N_A = 6.02 \times 10^{23}$  is Avogadro's number,  $\rho_m$  is the melt density,  $M$  is the molar mass of

the volatile species, and  $X$  is the mass fraction of dissolved volatiles.

[23] The presence of impurities, such as crystals, with lower interfacial energy between bubble (i.e., vapor) and solid (i.e., crystal) than the interfacial energy between vapor and liquid (i.e., melt), decreases the Gibbs free energy of a stable bubble nucleus. Consequently, higher nucleation rates can be achieved at a given supersaturation. Nucleation in the presence of such nucleation sites is called heterogeneous nucleation, as opposed to homogeneous nucleation. All else being equal,  $\Delta P_{s,\text{hetero}} < \Delta P_{s,\text{homo}}$  and within the context of classical nucleation theory is a consequence of  $\psi < 1$ . The theoretical value of  $\psi$  is determined by the contact angle  $\theta$  between nucleation sites and exsolved vapor phase (Figure 2)

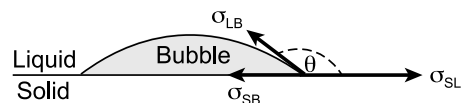
$$\psi = \frac{(2 - \cos \theta)(1 + \cos \theta)^2}{4}. \quad (9)$$

The wetting angle is controlled by the relative values of the interfacial (surface) tensions: vapor-liquid ( $\sigma_{\text{LB}}$ ), vapor-solid ( $\sigma_{\text{SB}}$ ), and solid-liquid ( $\sigma_{\text{SL}}$ ). It is defined as

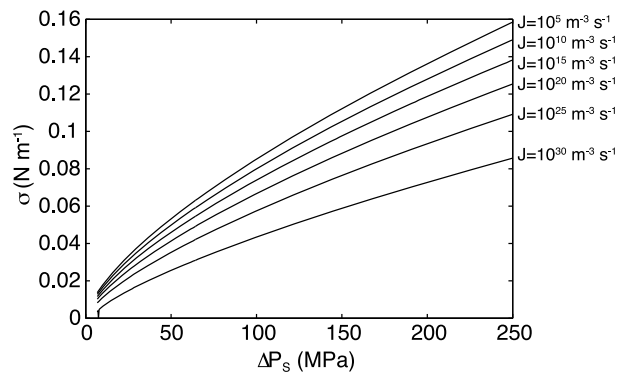
$$\cos \theta = \frac{\sigma_{\text{SB}} - \sigma_{\text{SL}}}{\sigma_{\text{LB}}}. \quad (10)$$

For  $\theta < 90^\circ$  the bubble is poorly wetting, as opposed to for  $\theta > 90^\circ$ . For  $\theta = 0^\circ$  the value of  $\psi = 1$  and the presence of crystals has no effect on nucleation.  $\psi$  also depends on the shape of the nucleation substrate, where Equation (9) corresponds to the case of a planar substrate. Relative to a planar surface, the nucleation efficiency will be enhanced on a concave substrate, but reduced on a convex substrate [Cluzel et al., 2008, and references therein].

[24] In many natural and artificial systems nucleation proceeds at significantly higher rates than would be predicted by classical nucleation theory. A number of explanations have been proposed to explain this discrepancy, including the presence of submicroscopic impurities or inhomogeneities to reduce the energy barrier for nucleation [e.g., Lubetkin, 2003]. To what extent bubble nucleation in erupting



**Figure 2.** Schematic diagram illustrating the definition of contact angle,  $\theta$ , and interfacial tension between solid and bubble,  $\sigma_{\text{SB}}$ , liquid and bubble,  $\sigma_{\text{LB}}$ , as well as solid and liquid,  $\sigma_{\text{SL}}$ . If the volatiles within the bubble are more wetting to the solid (crystal) than the liquid (melt), then  $\theta > 90^\circ$ .



**Figure 3.** Contours of predicted bubble nucleation rates (black lines) for Novarupta dacite at 850°C, for different supersaturation pressures,  $\Delta P_s$ , as well as corresponding dissolved H<sub>2</sub>O content and H<sub>2</sub>O diffusivity [Zhang and Ni, 2010] and surface tension,  $\sigma$ .

magmas occurs at higher rates than would be predicted from classical nucleation theory, as seems to be the case for many other nucleation processes, is an important question [e.g., Mourtada-Bonnefoi and Laporte, 2004]. Thus, empirical values of surface tension for natural silicate melts may perhaps constitute an *effective* surface tension

$$\sigma_e = \sigma \psi^{1/3}, \quad (11)$$

with  $\psi$  accounting not only for the presence of crystals, but any other effect that reduces Gibbs free energy of a stable bubble nucleus.

## 4.2. Bubble Nucleation During Magma Ascent

[25] During rapid magma ascent and decompression, the melt may become increasingly supersaturated, if volatiles cannot diffuse fast enough from the melt into bubbles [e.g., Lensky et al., 2004]. However, even if diffusion is fast enough, the melt may become supersaturated if bubbles cannot grow fast enough, relative to the rate of magma ascent and decompression. In this case the pressure of exsolved volatiles within bubbles remains high. The ability of volatiles to diffuse and exsolve into bubbles depends on the concentration of dissolved volatiles at the melt-vapor interface, which in turn depends on the pressure of the exsolved volatiles within bubbles. Therefore, if bubbles cannot grow fast enough, the resultant overpressure prevents volatile exsolution, resulting in supersaturation [e.g., Gonnermann and Manga, 2007].

[26] Classical nucleation theory predicts a continuous and steep increase in nucleation rate as  $\Delta P_s$  increases,

with nucleation rates of up to  $\sim 10^{30}$  bubbles  $m^{-3} s^{-1}$  for values of  $\Delta P_s \sim 100$  MPa (e.g., Figure 3) [Mangan and Sisson, 2005]. To what extent continuous nucleation is substantiated by experiments, where in some cases it appears that nucleation occurs as a discrete event, is unclear [e.g., Gardner et al., 1999; Mourtada-Bonnefoi and Laporte, 1999, 2004; Gardner and Ketcham, 2011]. Models of continuous nucleation predict a steep increase in nucleation rate within a very short time interval [e.g., Toramaru, 1989, 1995], perhaps experimentally indistinguishable from a discrete event.

[27] During the steep increase in nucleation rate associated with rapid decompression and increasing supersaturation, the spatial density of bubbles will reach a threshold where all of the melt begins to be affected by volatile diffusion into bubbles. Because  $J$  is so strongly dependent on supersaturation, it will rapidly drop to zero once this threshold is reached [e.g., Toramaru, 1989, 1995; Mourtada-Bonnefoi and Laporte, 2002]. How soon after the onset of nucleation this threshold is reached depends on the relative rates of diffusion and decompression. The former scales as the ratio of volatile diffusivity to the square of the thickness of melt between individual bubbles, which decreases as the number and size of bubbles increases. Consequently, the rates of bubble growth assert a strong control on the time interval over which bubbles can nucleate.

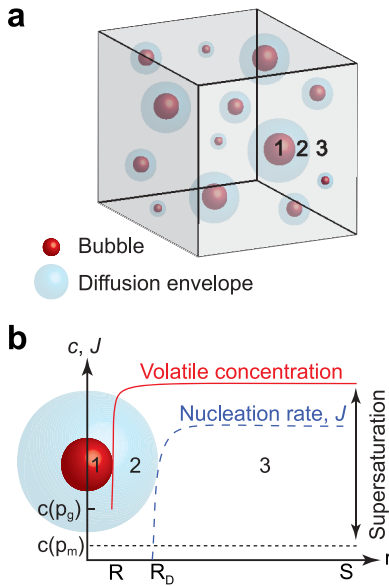
[28] As depicted in Figure 4, the entire process may be envisaged by a volume of magma that consists of fully saturated melt, bubbles and surrounding diffusion envelopes [Kedrinisky, 2009]. The threshold for the cessation of bubble nucleation can be viewed as the time,  $\tau_n$ , at which the entire melt volume is encompassed by diffusion envelopes [Toramaru, 1989, 1995; Kedrinisky, 2009]. Within this conceptual framework, and neglecting bubble coalescence,

$$N_m = \int_0^{\tau_n} J dt_n, \quad (12)$$

where  $t_n$  is the time since the onset of nucleation.

## 4.3. The Moments of the Bubble Size Distribution

[29] One approach to modeling  $N_m$  has been to solve the equation of motion about moments of the bubble size distribution, assuming that there is no coalescence or breakage of bubbles [e.g., Randolph and Larson, 1971; Toramaru, 1989, 1995]. This



**Figure 4.** (a) Schematic representation of the proposed bubble nucleation and growth model [e.g., *Kedrinskiy*, 2009, Figure 10]. (1) represents the bubble; (2) represents the diffusion envelope that is the part of the melt where volatile concentrations and correspondingly nucleation rates are low; and (3) is the nucleation region. (b) Schematic graph of volatile concentration in the melt,  $c$ , as a function of radial distance,  $r$ . The concentration,  $c(p_g)$ , at the melt-vapor interface ( $r = R$ ) is dependent on the pressure of the exsolved volatiles (gas) inside the bubble,  $p_g$ . The difference between the actual volatile concentration (solid red) and the equilibrium saturation concentration,  $c(p_m)$ , at ambient pressure,  $p_m$ , is the supersaturation. Nucleation rate (long-dashed blue),  $J$ , is negligible inside the diffusion envelope of radius  $r = R_D$ . The radius of the melt shell associated with a given bubble is denoted as  $r = S$  and nucleation ceases when the radius of the diffusion envelope approaches a value of  $R_D \rightarrow S$ .

method has been employed to assess magma decompression rates and we summarize it here to provide the necessary background for the subsequent discussion on modeling of VSDs via diffusive bubble growth.

[30] The zeroth moment,  $M_0$  of the BSD equals  $N_m$  and is calculated as

$$\frac{M_0}{dt} = \frac{N_m}{dt} = J. \quad (13)$$

The first moment,  $M_1$ , is given by

$$\frac{M_1}{dt} = GM_0, \quad (14)$$

where  $G$  is the growth rate of a bubble with mean radius  $\bar{R} = M_1/M_0$ . Higher order moments are calculated from

$$\frac{M_j}{dt} = jGM_{j-1}. \quad (15)$$

[31] The zeroth and first order moments have been the primary focus of previous studies [*Toramaru*, 1989, 1995, 2006]. In addition to Equations (6), (13) and (15), the zeroth and first order moments require an equation for the average bubble growth rate  $G = d\bar{R}/dt$ . Bubble growth is governed by mass conservation of volatiles and momentum balance of the melt-fluid interface. The latter, assuming constant melt viscosity,  $\eta$ , and a spherical bubble geometry, is given by

$$\frac{d\bar{R}}{dt} = \frac{\bar{R}}{4\eta} \left( p_g - p_m - \frac{2\sigma}{\bar{R}} \right), \quad (16)$$

where  $p_g$  is the gas pressure inside bubbles and  $p_m$  the pressure in the surrounding melt.

[32] Typically,  $p_m$  decreases at some prescribed rate,  $dp_m/dt$ , whereas  $p_g$  is calculated from conservation of mass

$$\frac{d}{dt} (p_g \bar{R}^3) = 4 \frac{BT}{m} \bar{R}^2 \rho_m D \left( \frac{\partial c_w}{\partial r} \right)_{r=\bar{R}}. \quad (17)$$

Here  $B$  is the universal gas constant,  $T$  is absolute temperature,  $m$  is the molar mass of the volatile species,  $\rho_m$  is the density of the melt,  $r$  is the radial coordinate with  $r = 0$  at the bubble center, and  $c_w$  is the concentration of dissolved  $H_2O$ . The diffusive flux of volatiles from the melt into the bubble wall, which can be obtained from the mean field approximation [*Toramaru*, 1989]

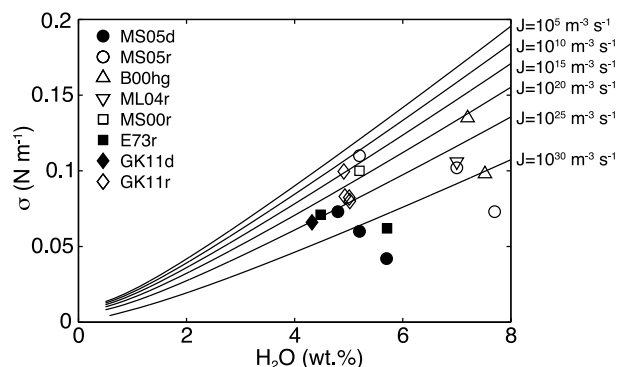
$$\left( \frac{\partial c_w}{\partial r} \right)_{r=\bar{R}} \approx \frac{c_{w,0} - c_{w,p_g}}{\bar{R}}. \quad (18)$$

Here  $c_{w,0}$  is the initial volatile content and  $c_{w,p_g}$  is the equilibrium volatile content at pressure  $p_g$ , obtained from a suitable solubility formulation.

#### 4.4. The Role of Surface Tension, $\sigma$

[33] The advantage of solving Equations (13)–(15), together with Equations (16) and (17), lies in the efficiency of the mean field approximation (Equation (18)), albeit at the cost of potential inaccuracies. It also requires explicit calculation of  $J$  in Equation (13), which requires that  $\sigma$  is known. However, because of the exceedingly strong dependence





**Figure 5.** Experimentally determined values of surface tension,  $\sigma$ , for rhyolitic (open symbols) and dacitic (filled symbols) melts at different dissolved  $\text{H}_2\text{O}$  contents. Also shown are contours of predicted bubble nucleation rates (black lines) for a Novarupta dacite at  $850^\circ\text{C}$ , different dissolved  $\text{H}_2\text{O}$  content and  $\text{H}_2\text{O}$  diffusivity calculated from the formulation of Zhang and Ni [2010]. Most experimentally determined values of  $\sigma$  are at high water content and, under the assumption of classical nucleation theory, would predict very high nucleation rates,  $J$ . Moreover, uncertainties in  $\sigma$  by only a few percent result in predictions of  $J$  that differ by several orders in magnitude. Experimental values are by Mangan and Sisson [2005] (rhyolite: MS05r, dacite: MS05d), Bagdassarov et al. [2000] (haplogranite: B00hg), Mourtada-Bonnefoi and Laporte [2004] (rhyolite: ML04r), Mangan and Sisson [2000] (rhyolite: MS00r), Epel'baum et al. [1973] (rhyolite: E73r) and Gardner and Ketcham [2011] (rhyolite: GK11r, dacite: GK11d).

of  $J$  on  $\sigma$  (Equation (6)), relatively modest uncertainties in  $\sigma$  can significantly affect model predictions (Figure 3).

[34] That there is considerable uncertainty in  $\sigma$  is shown in Figure 5, which includes a compilation of the empirical values of  $\sigma$  for rhyolitic and dacitic melts; note that these are determined almost exclusively at water contents above 4 weight percent (wt.%). Because of the tradeoff between  $\Delta P_s$  and  $\sigma$  (Figure 3), uncertainties in  $\sigma$ , especially at low  $\text{H}_2\text{O}$ , will translate into significant uncertainties in  $\Delta P_s$  and in predicted decompression rates via the method of moments. This problem may be further exacerbated by the limitations associated with classical nucleation theory itself, which for many different nucleation processes tends to under-predict nucleation rates [e.g., Lubetkin, 2003].

[35] In contrast to bubble nucleation, bubble growth is relatively insensitive to  $\sigma$ , if bubbles are of radius  $>10^{-6}$  m (Figure 6), the lower bound for vesicles preserved in many pyroclasts, including those from Novarupta [Adams et al., 2006]. This fact

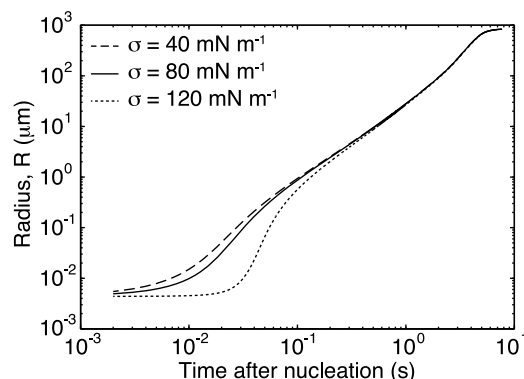
provides the main motivation for our methodology of estimating  $J$  and  $dp_m/dt$  indirectly from calculations of diffusive bubble growth, which furthermore do not employ the mean field approximation.

## 5. Conceptual Framework for Modeling Bubble Size Distributions

[36] The monotonic or near-monotonic increase in vesicle abundance with decreasing size of the Novarupta VSDs [Adams et al., 2006] can be interpreted to be produced during a period of continuous bubble nucleation and growth, during which supersaturation and, hence,  $J$  continually increased [Toramaru, 1989, 1995; Massol and Koyaguchi, 2005; Nowak et al., 2011]. Within this conceptual framework, the smaller a bubble, the younger it is [e.g., Yamada et al., 2008] and its growth time,  $\tau$ , can be obtained from diffusive bubble-growth simulations, where  $\tau(a)$  is the time for each of  $n(a)$  vesicles to grow to size  $a$  after nucleation. We shall assume that bubble growth ends at time  $t_f$ , and that this time is the same for all bubbles. Therefore, a bubble nucleated time  $t = t_f - \tau_a$ . It is thus possible to transform  $N(a)$  to  $N(t)$  and to estimate the nucleation rate as

$$J(t) = \frac{dN(t)}{dt}. \quad (19)$$

[37] Assuming that the maximum nucleation rate occurs when the maximum degree of supersaturation



**Figure 6.** Modeled bubble radius,  $R$ , as a function of time after nucleation for constant values of surface tension,  $\sigma$ . Initial dissolved  $\text{H}_2\text{O}$  and  $\text{CO}_2$  are 2.62 wt.% and 34 ppm, respectively. Melt composition is Novarupta dacite, temperature is  $850^\circ\text{C}$ , diffusivity is calculated using the formulation of Zhang and Ni [2010], solubility is calculated using the formulation of Liu et al. [2005], and viscosity using the formulation of Hui and Zhang [2007].

is reached, the pre-eruptive pressure of magma storage,  $P_{\text{ch}}$ , provides an upper bound on the supersaturation pressure,  $\Delta P_s$ . Together with the maximum nucleation rates and Equation (6) it is thus possible to obtain a lower bound on  $\sigma_e$ . Assuming a constant  $\sigma_e$  permits the calculation of  $\Delta P_s(t)$  from  $J(t)$  and subsequently a minimum decompression rate

$$\frac{dp_m}{dt} = \frac{d}{dt}[\Delta P_s(t)]. \quad (20)$$

## 6. Diffusive Bubble Growth Modeling

### 6.1. Spatial Distribution of Bubbles

[38] Conceptually we assume that the vesicle size distribution,  $n(a)$ , is the consequence of bubbles nucleating throughout the time interval  $0 \leq t \leq \tau_n$  and at random locations within the supersaturated melt that is not encompassed by diffusion envelopes (Figure 4). Once a bubble is nucleated it starts growing and its time of nucleation,  $\tau_f - \tau_a$ , and final bubble radius,  $R = a$ , are inversely correlated. That is, the oldest bubbles have the largest radii.

[39] For simplicity, we assume that the final distribution,  $n(R) = n(a)$ , approximates a smoothly varying concentration of dissolved volatiles throughout the melt. Given the spherical approximation of bubble growth [e.g., *Amon and Denson*, 1984; *Arefmanesh and Advani*, 1991; *Proussevitch et al.*, 1993], this corresponds to the condition that the volatile concentration at the outer radius of the melt shell,  $c_{r=S}$ , has the same value for all bubbles. The resultant spatial distribution of bubble cells would be space-filling and similar to an Apollonian packing geometry [*Borkovec et al.*, 1994; *Blower et al.*, 2002].

### 6.2. Bubble Nucleus

[40] We assume that the size of the critical bubble nucleus can be derived from the Laplace relation, which describes the mechanical equilibrium condition for a bubble

$$p_g - p_m = \frac{2\sigma}{R}. \quad (21)$$

The critical bubble radius,  $R_c$ , that satisfies Equation (21) is given by [e.g., *Proussevitch et al.*, 1993]

$$R_c = \frac{2\sigma}{\Delta P_s}. \quad (22)$$

If  $R < R_c$  the bubble will shrink and disappear. To assure bubble growth, the initial bubble therefore has to satisfy the condition  $R > R_c$  [e.g.,

*Proussevitch et al.*, 1993] and we use an initial bubble radius of  $R = (1 + \gamma) R_c$ , with  $\gamma = 10^{-2}$ .

### 6.3. Diffusive Bubble Growth

[41] Bubble growth is a consequence of magma decompression, due to a decrease in ambient pressure,  $p_m$ . This decrease in ambient pressure causes volume expansion of the already exsolved volatiles and the further diffusion of volatiles from the melt into existing bubbles. Here we consider both  $\text{H}_2\text{O}$  and  $\text{CO}_2$  as the exsolving volatile species. The solubilities of  $\text{H}_2\text{O}$  and  $\text{CO}_2$ , denoted as  $s_w$  and  $s_c$ , respectively, are based on the formulation by *Liu et al.* [2005] and represent a reasonable approximation for joint  $\text{H}_2\text{O}$ - $\text{CO}_2$  solubilities in dacite melts [*Zhang et al.*, 2007].  $s_w$  and  $s_c$  depend on temperature and their partial pressure within the coexisting gas phase,  $(1 - x_c) p_g$  and  $x_c p_g$ , respectively. Here  $x_c$  is the mole fraction of exsolved  $\text{CO}_2$  within the bubble.

#### 6.3.1. Volatile Exsolution

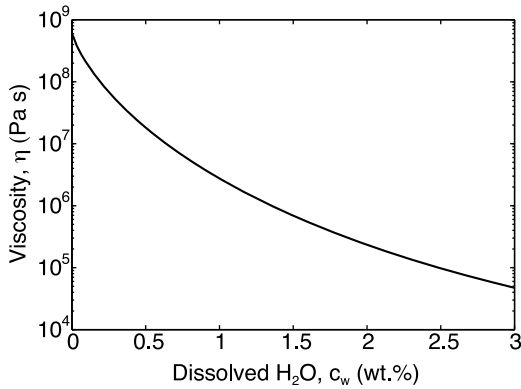
[42] The mass flux of each volatile species into a bubble is determined by the concentration gradient in the melt at  $r = R$ , the melt-gas interface

$$q_i = D_i \left( \frac{\partial c_i}{\partial r} \right)_{r=R}. \quad (23)$$

Here  $c_i$  is the concentration of volatile species  $i$  ( $i = w$  for  $\text{H}_2\text{O}$  or  $i = c$  for  $\text{CO}_2$ ).  $D_i$  is the diffusivity of the given species within the melt, based on Equations 20 and 41 of *Zhang and Ni* [2010] for  $\text{H}_2\text{O}$  and  $\text{CO}_2$ , respectively. The concentration of dissolved  $\text{H}_2\text{O}$  and  $\text{CO}_2$  at  $r = R$  are assumed to be equilibrium solubilities at pressure  $p_g$  and vapor phase composition  $x_c$  [*Liu et al.*, 2005]. As indicated by Equation (22), newly nucleated bubbles are under considerable internal pressure ( $p_g - p_m \sim 10^7$  Pa), because of the Laplace pressure,  $2\sigma/R$  [*Toramaru*, 1989, 1995]. During the early stages of bubble growth, the associated decrease in Laplace pressure asserts a strong influence on volatile exsolution, given that  $s_w$  and  $s_c$  are strongly pressure dependent [*Liu et al.*, 2005]. This potentially rapid change in  $s_w$  and  $s_c$  results in steep concentration gradients and large fluxes,  $q_i$ , of exsolving volatiles.

#### 6.3.2. Volatile Diffusion

[43] The mass flux of exsolving volatiles is calculated from a diffusion model for volatiles within the melt surrounding each bubble. We employ the cell model for bubble growth [e.g., *Amon and Denson*,



**Figure 7.** Viscosity,  $\eta$ , as a function of dissolved water,  $c_w$ , for Novarupta dacite, based on the formulation of Hui and Zhang [2007] at a temperature of 850°C.

1984; Arefmanesh and Advani, 1991; Proussevitch *et al.*, 1993], wherein bubbles are assumed to be spatially distributed such that each bubble can be approximated as a sphere surrounded by a spherical melt shell of thickness  $S - R$ . Because of the spherical symmetry inherent in this approximation, volatile diffusion simplifies to

$$\frac{\partial c_i}{\partial t} + v_r \frac{\partial c_i}{\partial r} = \frac{1}{r^2} \frac{\partial c_i}{\partial r} \left( D_i r^2 \frac{\partial c_i}{\partial r} \right), \quad (24)$$

where  $v_r = dR/dt$  is the radial velocity of melt at  $r = R$ . Equation (24) is solved using an implicit finite difference scheme in a Lagrangian frame of reference and with a non-uniform grid. The latter assures that concentration gradients at the melt-vapor interface are accurately resolved [Arefmanesh and Advani, 1991; Proussevitch *et al.*, 1993]. The boundary condition at  $r = S$  is

$$\left( \frac{\partial c_i}{\partial r} \right)_{r=S} = 0, \quad (25)$$

and as mentioned previously,

$$(c_i)_{r=R} = s_i(p_g, x_c). \quad (26)$$

### 6.3.3. Mass Conservation

[44] Mass conservation of volatiles requires that

$$\frac{d}{dt} (\rho_g R^3) = 4R^2 \rho_m \sum_i q_i, \quad (27)$$

where the density of the exsolved gas phase,  $\rho_g$ , depends on  $p_g$  via an equation of state [Kerrick and Jacobs, 1981] and melt density,  $\rho_m = 2400 \text{ kg m}^{-3}$ , [Spera, [2000] is approximated as constant. We assume that each bubble grows as a closed system.

In other words, no gas is lost from the bubble. This approximation is valid if the timescale for bubble growth is much shorter than the characteristic timescale for permeable flow through interconnected bubbles. We provide an assessment of this assumption in section 8.5.

### 6.3.4. Momentum Balance

[45] Bubbles grow as a consequence of volatile flux into bubbles and volume expansion of the already exsolved volatiles. The balance between volume expansion and additional exsolution of volatiles determines how much  $p_g$  and, hence,  $\rho_g$  decrease. Bubble growth is resisted by viscous and capillary stresses, which are balanced by the pressure difference between the gas mixture inside the bubble and the surrounding melt

$$p_g(t) - p_m(t) = \frac{2\sigma}{R} + 4\eta_e \frac{1}{R} \frac{dR}{dt}. \quad (28)$$

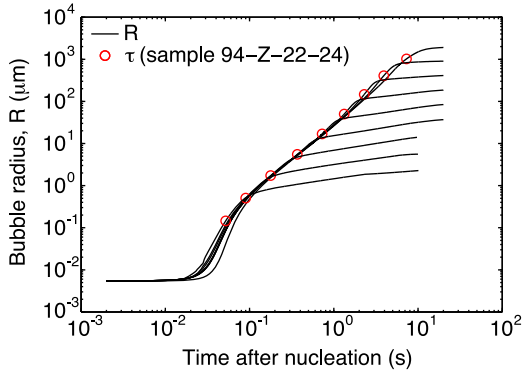
Here  $\eta_e$  is the effective viscosity accounting for the radially variable H<sub>2</sub>O-dependent viscosity (Figure 7) [Lensky *et al.*, 2001; Hui and Zhang, 2007] and  $dR/dt$  is the radial velocity of the melt-vapor interface. The pressure in the surrounding melt,  $p_m$ , is a boundary condition and may be time-dependent.

### 6.3.5. Solution Method

[46] The coupled Equations (24)–(28) are solved using a similar formulation as described by Arefmanesh and Advani [1991] and by Proussevitch *et al.* [1993]. However, in our case Equation (24) has to be solved separately for H<sub>2</sub>O and CO<sub>2</sub>, but with concentration boundary conditions, that couple both diffusion calculations. At each time step a consistent solution for Equations (24)–(28) is obtained through nonlinear minimization of the difference between gas volume predicted by Equations (27) and (28). Because we model bubble growth over a range of bubble sizes spanning up to 6 orders of magnitude, the nonlinear grid-spacing parameter for Equation (24), discussed in detail by Proussevitch *et al.* [1993] and by Chouet *et al.* [2006], is optimized to maintain good mass balance of H<sub>2</sub>O and CO<sub>2</sub> throughout the calculation.

## 7. Modeling the Bubble Size Distribution

[47] We focused our study on the three aforementioned clasts from Episode III of the 1912 Novarupta eruption (section 2.5), because they represent a range



**Figure 8.** Modeled bubble radius,  $R$ , as a function of time after bubble nucleation, for bubbles with different-size diffusion envelopes ranging from  $v_m = 1 \mu\text{m}$  (smallest  $R$ ) to  $v_m = 10^8 \mu\text{m}$  (largest  $R$ ) in factors of 10.

in clast vesicularity and afford the opportunity to investigate potential differences in the vesiculation process within a sample.

### 7.1. Initial Conditions

[48] Initial conditions were based on the experimental work by *Hammer et al.* [2002], who determined pre-eruptive volatile saturation and pressure using mineral phase relations. They obtained a pre-eruptive  $\text{H}_2\text{O}$  saturation pressure for Novarupta dacite of  $p_{\text{H}_2\text{O}} = 50 \text{ MPa}$  at a temperature of  $850^\circ\text{C}$ . Natural mineral and melt compositions were also reproduced under saturation conditions with a mixed  $\text{H}_2\text{O} + \text{CO}_2$  fluid for the same  $p_{\text{H}_2\text{O}}$ , if the fluid phase consists of up to 10%  $\text{CO}_2$  ( $x_c \leq 0.1$ ). We therefore used initial conditions that are equivalent to  $p_{\text{H}_2\text{O}} = 50 \text{ MPa}$  and  $x_c = 0.1$  using the solubility model of *Liu et al.* [2005].

[49] The initial concentrations of dissolved  $\text{H}_2\text{O}$  and  $\text{CO}_2$  are  $c_{(w,0)} = 2.62 \text{ wt.}\%$  and  $c_{(c,0)} = 34 \text{ ppm}$ , respectively. The corresponding equilibrium solubility pressure is  $p_{\text{sol}} = 55.52 \text{ MPa}$ . We used an initial bubble radius of  $R_0 = (1 + \gamma) 2\sigma/p_{\text{sol}}$  (section 4) at an internal pressure  $p_g = p_{\text{sol}}$ , where  $\gamma = 10^{-2}$ . We assumed a constant value of  $\sigma = 0.08 \text{ N m}^{-1}$  [*Gardner and Ketcham*, 2011, and references therein], because model results are not significantly sensitive to  $\sigma$ , within the range of measured values (Figures 5 and 6). Furthermore, we assumed a constant ambient pressure,  $p_m = 10^5 \text{ Pa}$ , for the bubble growth calculations.

### 7.2. Modeling the Volume Fraction Density, $\phi(a_i)$

[50] Using these initial and boundary conditions, we modeled bubble growth for individual bubbles,

$j$ , each of which is surrounded by a given volume of melt,  $v_{m,j}$ . Each bubble undergoes a distinct growth history, beginning at time  $t = t_f - \tau_j$  (Figure 8) and resulting in a final vesicle size,  $R_j(\tau_j)$ , after a growth time,  $\tau_j$ . The values of  $\tau_j$  and  $R_j(\tau_j)$  were determined from the condition that

$$c_{(w,r=S)} = (1 - \epsilon) c_{(w,0)}, \quad (29)$$

where  $\epsilon \ll 1$  assures that the condition of Equation (29) represents the threshold at which diffusion begins to affect the entire volume of melt, that is when  $J$  begins to decrease rapidly (Figures 8 and 9). Furthermore, the requirement that  $c_{(w,r=S)}$  is the same for all bubbles assures that the concentration of  $\text{H}_2\text{O}$  between any two bubbles is continuously smooth. Because the diffusivity of  $\text{CO}_2$  is considerably smaller than  $\text{H}_2\text{O}$  diffusivity, we found that  $c_{(c,r=S)} = c_{(c,0)}$  for all of our calculations (Figure 10).

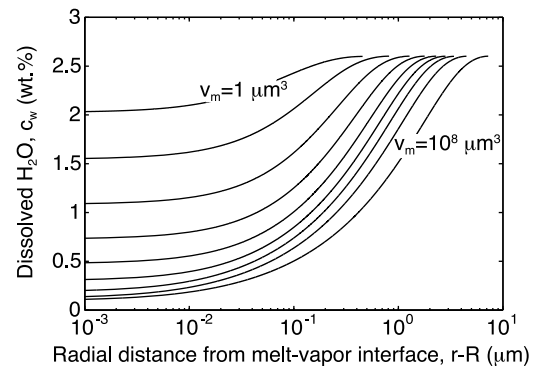
[51] Using smooth cubic spline interpolation of the discrete values  $R_j = f(\tau_j)$ , we obtained the functions  $R = f_R(\tau)$  (Figure 11), and from the relation

$$v_{m,j} = \frac{4\pi}{3} [S_j(t)^3 - R_j(t)^3], \quad (30)$$

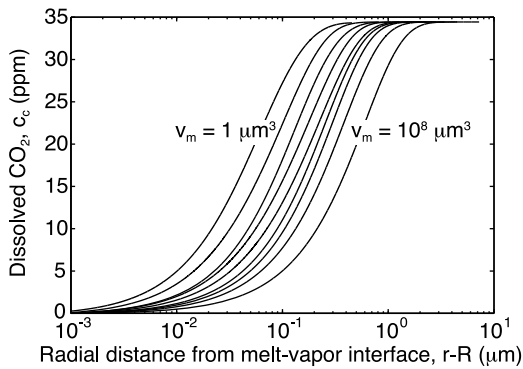
we similarly obtained  $S = f_S(\tau)$ .

[52] We then find the values of  $\tau = \tau_i$ , corresponding to the condition  $f_R(\tau_i) = a_i$ . Defining  $R_i = f_R(\tau_i)$  and  $S_i = f_S(\tau_i)$ , we calculated the predicted volume fraction of vesicles (Figure 12) from

$$\phi_{\text{mod}} = \sum_i \phi_{\text{mod}}(a_i) = \sum_i \left[ n(a_i) \left( \frac{R_i}{S_i} \right)^3 \right]. \quad (31)$$



**Figure 9.** Concentration of dissolved  $\text{H}_2\text{O}$  in weight percent (wt.%) at time  $\tau_j$  after nucleation, for bubbles with  $v_{m,j} = 10^0, 10^1, \dots, 10^8 \mu\text{m}^3$  as a function of radial distance from the melt-vapor interface,  $r-R$ , of the growing bubble.

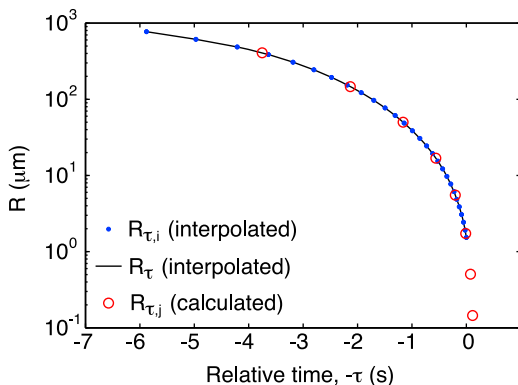


**Figure 10.** Concentration of dissolved CO<sub>2</sub> in parts per million (ppm) at time  $\tau_j$  after nucleation, for bubbles with  $v_{m,j} = 10^0, 10^1.. 10^8 \mu\text{m}^3$  as a function of radial distance from the melt-vapor interface,  $r-R$ , of the growing bubble.

[53] We repeated the steps outlined in this section iteratively for different values of  $\epsilon$  until the misfit between modeled and measured volume fraction,  $|\phi_{\text{mod}} - \phi_{\text{sam}}|$  was minimized. We found that  $10^{-3} \leq \epsilon \leq 10^{-2}$  provides a good fit (Figure 13). The resultant volume fraction density distributions,  $\phi_{\text{mod}}(a_i)$ , together with the actual distributions,  $\phi_{\text{sam}}(a_i)$ , are shown in Figure 12.

### 7.3. Nucleation Rate, $J(t)$ , and Pressure, $p_m(t)$

[54] Using the functional relation  $f_R(\tau)$  at values of  $R = a_i$  and a smooth cubic spline interpolation, we obtain  $N(t) = N(t_f - \tau)$  from the discrete distribution  $N(a_i)$ . We then calculate the time-dependent nucleation rate,  $J(t)$ , from  $N(t)$  using Equation (19). The results are shown in Figure 14.



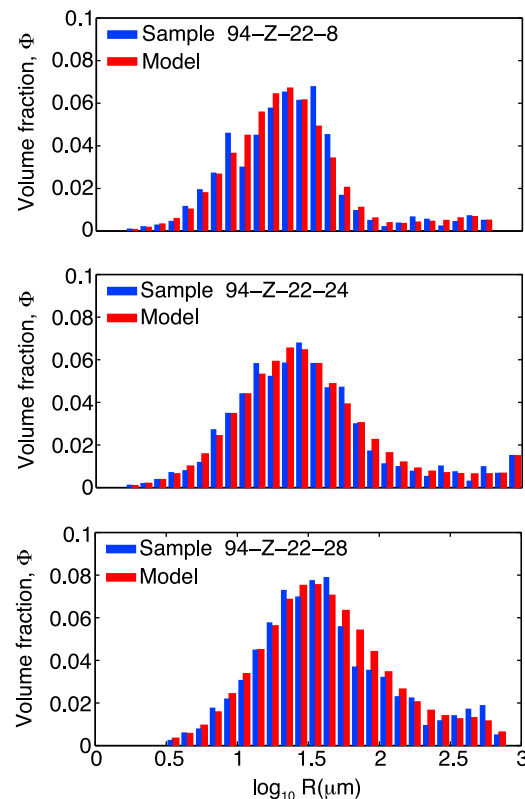
**Figure 11.** Modeled and interpolated values of  $R_j$ ,  $R_i$  and  $R$  versus  $\tau_j$ ,  $\tau_i$  and  $\tau$ , respectively (sample 94-Z-22-24).

[55] Assuming that the maximum value of  $J(t)$  occurs at  $\Delta P_s = P_{\text{ch}}$ , we calculate  $\sigma_e$  using Equation (6) and subsequently  $\Delta P_s(t)$  from  $J(t)$  using Equation (20). The results are shown in Figure 15.

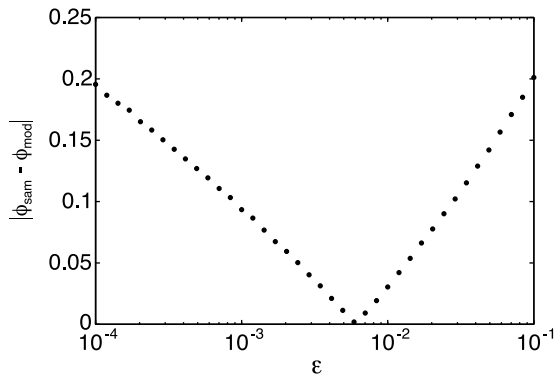
## 8. Discussion

### 8.1. Nucleation and Decompression Rates

[56] Modeling of vesicle size distributions measured in Novarupta dacite clasts 94-Z-22-8, 94-Z-22-24 and 94-Z-22-28 (Figure 12) provides constraints on nucleation rate (Figure 14) and decompression rate (Figure 15). The three clasts span a range in vesicularity ( $\phi = 0.52, 0.64$  and  $0.75$ ) within a single tephro-stratigraphic level, interpreted in terms of radial variability within the conduit [Adams *et al.*, 2006]. We find that the observed textural variability requires approximately 25% variability ( $\approx 1 \text{ MPa s}^{-1}$ ) in decompression rate. The estimated peak nucleation rates are similar to the predictions by Toramaru [1989, 1995, 2006], but at decompression rates of  $< 10^7 \text{ Pa s}^{-1}$ , which are considerably lower than those predicted by Toramaru [1989,



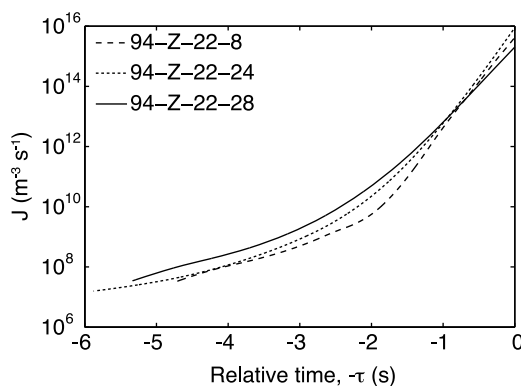
**Figure 12.** Comparison of modeled ( $\phi_{\text{mod}}(a_i)$ ) and actual ( $\phi_{\text{sam}}(a_i)$ ) distributions of vesicle volume fraction of Novarupta samples 94-Z-22-8, 24 and 28 ( $\phi = 0.52, 0.64$  and  $0.75$ , respectively).



**Figure 13.** Misfit  $|\phi_{\text{mod}} - \phi_{\text{sample}}|$  for sample 94-Z-22-24 as a function of the parameter  $\epsilon$  (Equation (29)).

1995, 2006] and more consistent with conduit flow models [e.g., *Koyaguchi, 2005; Massol and Koyaguchi, 2005*].

[57] The model makes no assumptions about where or when bubble nucleation takes place, relative to fragmentation. We find that the peak in nucleation coincides with a peak in volumetrically averaged bubble overpressure and values that are approximately equal to empirical overpressures at which fragmentation is predicted (see section 8.2 for a detailed discussion). Unless fragmentation occurred by some other mechanism than postulated by current fragmentation models [e.g., *Spieler et al., 2004a; Mueller et al., 2008*], the most plausible explanation would be that the predicted coincident peaks in bubble overpressure and nucleation also coincided with fragmentation. In other words, bubble nucleation and fragmentation occur in very close temporal succession, suggesting that both feed back upon one another.



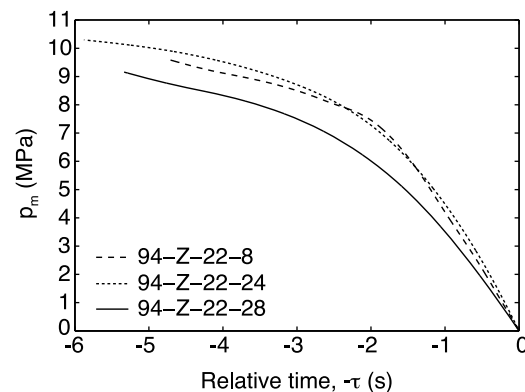
**Figure 14.** Predicted nucleation rates,  $J$ , as a function of  $-\tau$ .

## 8.2. Magma Fragmentation

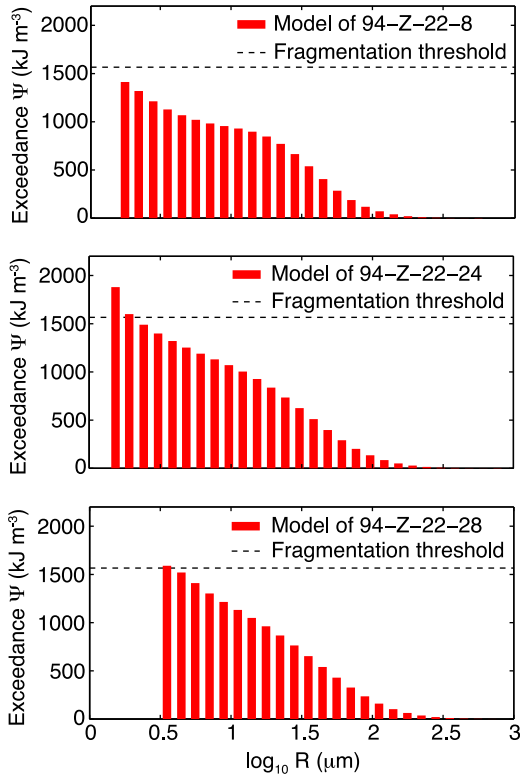
[58] The Novarupta dacite VSDs indicate magmatic conditions close to the critical stress threshold for magma fragmentation (Figure 16). The fragmentation stress threshold predicts brittle fracture of bubble walls, once  $(p_g - p_m)$  exceeds a critical value [e.g., *Alidibirov, 1994; Zhang, 1999; Spieler et al., 2004a; Mueller et al., 2008*]. This threshold, expressed in terms of a potential energy,  $\Psi_{\text{fr}}$ , increases with sample porosity,  $\phi$ , because the potential energy,  $\Psi$ , of the compressed volatiles within a given volume of magma increases with  $\phi$ . In addition, the thickness of the melt that surrounds individual bubbles, whose tensile strength must be exceeded for fragmentation, decreases with increasing  $\phi$ . However, as  $\phi$  increases, more bubbles become interconnected, thereby increasing permeability,  $k$ . Because permeable outgassing dissipates overpressure,  $\Psi_{\text{fr}}$  also depends on  $k$ , and has been determined empirically by *Mueller et al.* [2008] as

$$\Psi_{\text{fr}} = 8.21 \times 10^{11} \sqrt{k} + 1.54 \times 10^6 \quad \text{in units of } \text{J m}^{-3}. \quad (32)$$

[59] We find that the maximum volumetrically averaged bubble overpressure occurs when the model matches the vesicle size distributions preserved in the Novarupta pumice samples. If the associated potential energy was sufficient to cause magma fragmentation, magma permeability prior to fragmentation must have had values of  $k \ll 10^{-13} \text{ m}^2$  (dashed lines in Figure 16, see also discussion in section 8.5). Nucleation of the vast majority of bubbles took place within  $\sim 0.1$  s, perhaps not much longer than the time it takes for magma to traverse the fragmentation ‘level’ [*Spieler et al., 2004b*]. Consistent with the work by *Massol and Koyaguchi*



**Figure 15.** Predicted ambient pressure,  $p_m$ , as a function of  $-\tau$ .



**Figure 16.** The potential energy for fragmentation,  $\Psi$ , is calculated using Equation 14 of *Alidibirov* [1994] and the fragmentation threshold is based on the empirical formulation of *Mueller et al.* [2008] at a permeability of  $\ll 10^{-13} \text{ m}^2$ .

[2005], this implies that nucleation proceeded until fragmentation, and supports the notion that feedbacks between bubble nucleation and fragmentation are self-sustaining processes, with the peak in nucleation rate more or less coinciding with fragmentation.

### 8.3. Magma Ascent Rates

[60] Magma ascent rates below the depth of fragmentation,  $u$ , can be calculated from the estimated decompression rates,  $dp_m/dt$ , where

$$\frac{dp_m}{dt} = \frac{dz}{dt} \frac{dp_m}{dz} = u \frac{dp_m}{dz} = u \left( \frac{dp_m}{dz} \right)_\rho + u \left( \frac{dp_m}{dz} \right)_\eta. \quad (33)$$

Here,

$$\left( \frac{dp_m}{dz} \right)_\rho = (1 - \phi) \rho_m g \sim 10^4 \text{ Pa} \quad (34)$$

magma-static pressure loss and

$$\left( \frac{dp_m}{dz} \right)_\eta = 2(1 - \phi) \rho_m u^2 \frac{f}{d} \quad (35)$$

is the viscous pressure loss.  $d$  is the conduit diameter and  $f$  is the friction factor

$$f = \frac{16}{Re} + f_0. \quad (36)$$

$Re = ud\rho_m(1 - \phi)/\eta$  is the Reynolds number and  $f_0$  has values of about 0.002–0.02 [e.g., *Mastin and Ghiorso*, 2000]. Thus

$$\frac{dp_m}{dt} = u(1 - \phi) \rho_m g + 32u^2 \frac{\eta}{d^2} + 2u^3(1 - \phi) \rho_m \frac{f_0}{d}. \quad (37)$$

At the observed values of  $\phi$ , predicted values of  $dp_m/dt$  (Figure 15), likely values of  $d \sim 100 \text{ m}$  [*Hildreth and Fierstein*, 2012], as well as  $\eta > 10^5 \text{ Pa s}$  (Figure 7) [*Hui and Zhang*, 2007], predicted magma ascent rates just below the fragmentation depth are  $\sim 10 \text{ m s}^{-1}$ .

### 8.4. Bubble Coalescence

[61] The minimum bubble wall thickness in silicic melts, before spontaneous rupture and coalescence, is thought to be  $\sim 1 \mu\text{m}$  or less [e.g., *Klug and Cashman*, 1996; *Navon and Lyakhovskiy*, 1998]. Given a magma viscosity of  $\sim 10^5 \text{ Pa s}$  (Figure 7), the time required for capillary or gravitational thinning of bubble walls is too long to be of significance for the Novarupta dacites. Instead, coalescence is expected to be the consequence of bubble-wall thinning as a consequence of bubble growth (C. Nguyen et al. The lifetime of bubbles: film drainage and bubble coalescence, submitted to *Geochemistry, Geophysics, Geosystems*, 2012).

[62] Although we presently lack quantitative a-priori constraints on bubble coalescence rates during bubble growth, bubble shapes potentially provide a record thereof. After the melt film that separates adjacent bubbles ruptures and bubbles coalesce, capillary stress deforms bubbles back toward a spherical shape. This process is sometimes referred to as shape relaxation and occurs over a characteristic timescale [e.g., *Gardner*, 2007b]

$$\tau_{\text{relax}} \sim \frac{\eta R}{\sigma}. \quad (38)$$

[63] For Novarupta dacite with  $R \geq 10^{-6} \text{ m}$ , we find that  $\tau_{\text{relax}} \geq 1$ . It is therefore likely that textural evidence for coalescence should be preserved within pyroclasts [e.g., *Klug and Cashman*, 1996; *Gardner*, 2007b]. The VSDs modeled herein are based on a methodology whereby multiple coalesced vesicles with incomplete shape relaxation are treated as multiple individual vesicles [*Adams*



*et al.*, 2006]. Therefore, not modeling coalescence represents a reasonable approximation, especially for the smallest (i.e., youngest) vesicles.

### 8.5. Open-System Degassing

[64] The predicted onset of magma fragmentation requires the exsolution of only 10–20% of the pre-eruptive magmatic volatiles (Figures 9 and 10). We have measured approximately 2.4 wt.% H<sub>2</sub>O in the Novarupta dacite pumice samples, but it is unclear how much of this H<sub>2</sub>O is of magmatic origin, as opposed to from rehydration by atmospheric H<sub>2</sub>O (T. Giachetti and H. M. Gonnermann, Water in pumices: Rehydration versus incomplete degassing, submitted to *Earth and Planetary Science Letters*, 2012). Unless Novarupta pumice samples retain a large fraction of pre-eruptive magmatic volatiles within their matrix glass, degassing of most of these volatiles requires some degree of open-system gas loss, either before, during or after fragmentation. Open-system gas loss requires that coalescing bubbles form a continuous network of interconnected bubbles through which magmatic gas can flow. This is thought to occur at  $\phi \geq 0.3$  [e.g., *Rust and Cashman*, 2004; *Wright et al.*, 2009; *Rust and Cashman*, 2011]. Our bubble growth calculations assume a closed system (section 6.3.3). This is valid if there is insufficient time for gas to escape prior to magma fragmentation and will be substantiated in the subsequent paragraphs.

[65] An estimate for the rate of permeable gas flux can be obtained from Forchheimer's equation [*Forchheimer*, 1901; *Innocentini et al.*, 1998; *Rust and Cashman*, 2004]

$$\frac{p_g}{L} \sim \frac{\eta_g}{k_1 \rho_g} q_g + \frac{1}{k_2 \rho_g} q_g^2, \quad (39)$$

where it is assumed that gas flow is caused by a pressure difference of the order of  $p_g$  over a characteristic length of  $L$ . Here,  $q_g$  is the mass flux of gas,  $\eta \sim 10^{-5}$  Pa s is gas viscosity [e.g., *Takeuchi et al.*, 2009; *Rust and Cashman*, 2004, 2011],  $\rho_g = p_g m_g / R_g T$  is gas density,  $m_g \sim 10^{-2}$  kg mol<sup>-1</sup> is the molar mass of the gas phase,  $R_g = 8.314$  J mol<sup>-1</sup> K<sup>-1</sup> is the universal gas constant, and  $T$  is the temperature. In addition,  $k_1$  is the viscous (Darcian) permeability and  $k_2$  is the inertial (non-Darcian) permeability, which can be approximated as  $k_2 = 4.75 \times 10^{14} (k_1)^{1.87}$  [*Rust and Cashman*, 2004].

[66] The advective mass flux of dissolved volatiles, carried upward with the ascending magma at velocity  $u$ , is given by

$$q_m \sim u \rho_m (c_{(w,0)} + c_{(c,0)}). \quad (40)$$

For estimated magma ascent rates of  $u \sim 10$  m s<sup>-1</sup> and  $c_{(w,0)} + c_{(c,0)} \sim 10^4$  ppm, Equation (40) gives  $q_m \sim 10^2$  kg s<sup>-1</sup> m<sup>-2</sup>.

[67] A conservative requirement for open-system gas loss not to be negligible is that

$$q_g \geq 0.1 q_m. \quad (41)$$

For a relatively high value of  $k_1 \sim 10^{-12}$  m<sup>2</sup> [e.g., *Rust and Cashman*, 2004; *Wright et al.*, 2009; *Rust and Cashman*, 2011],  $m_g \sim 10^{-2}$  kg mol<sup>-1</sup>, the inequality of Equation (41) gives the minimum constraint that  $q_g \geq 10$  kg s<sup>-1</sup> m<sup>-2</sup>, which indicates that the last term in Equation (39) is dominant [*Degruyter et al.*, 2012], that is

$$\frac{\eta_g}{k_1 \rho_g} q_g \ll \frac{1}{k_2 \rho_g} q_g^2. \quad (42)$$

Consequently, from Equation (41)

$$\sqrt{\frac{p_g k_2 \rho_g}{L}} \geq 10 \text{ kg m}^{-2} \text{ s}^{-1}, \quad (43)$$

or

$$\sqrt{L} \leq \frac{p_g}{10^8 \text{ Pa m}^{-1/2}}. \quad (44)$$

[68] This inequality is difficult to satisfy below the fragmentation depth and the assumption of closed-system degassing to this point is therefore justified. Only within a very short distance of the 'fragmentation surface' will permeable gas loss become significant [*Mueller et al.*, 2008]. For example, at pressures near the fragmentation value, that is  $p_g \sim 10^7$  Pa, permeable gas loss becomes important within  $L \sim 10^{-2}$  m of the fragmentation surface or, equivalently, during the last  $\sim 10^{-3}$  s prior to fragmentation. It should be noted that  $k_1 \sim 10^{-12}$  m<sup>2</sup> is a conservatively high value and that values of  $k_1 \sim 10^{-13}$  m<sup>2</sup> are probably more reasonable to assume at fragmentation.

[69] Once the magma has fragmented,  $L \sim 10^{-2}$  m [e.g., *Rust and Cashman*, 2011; *Dufek et al.*, 2012], and Equation (39) indicates that open-system degassing of pyroclasts will be efficient over a wide range of values of  $p_g$ . Thus, unless the matrix glasses of the 1912 Novarupta dacite pyroclasts contain





a large fraction of the pre-eruptive volatiles, the majority of magmatic volatiles must have been lost after magma fragmentation and prior to quenching [e.g., Thomas *et al.*, 1994; Gardner *et al.*, 1996; Kaminski and Jaupart, 1997, 1998; Tait *et al.*, 1998; Xu and Zhang, 2002; Burgisser and Gardner, 2005; Takeuchi *et al.*, 2009], through combined volatile exsolution and open-system gas loss. If this is the case, and if open-system gas loss is sufficiently fast to balance exsolution, there may be negligible change in bubble size over viscous timescales of  $\eta R/\sigma \gtrsim 10$  s.

## 9. Conclusions

[70] Vesicle size distributions in pumice provide a direct record of the dynamics of magma ascent. For a given parcel of magma, ascent-driven decompression produces increasing volatile supersaturation and bubble nucleation rates. Older bubbles grow larger, because volatiles diffuse into the bubble from a larger volume of melt. Those portions of the melt not yet affected by diffusion into existing bubbles remain fully saturated and comprise a region where bubbles continue to nucleate. Once volatile concentrations begin to decrease throughout the entire melt volume, bubble nucleation rates diminish rapidly.

[71] The fundamental approximation of our modeling approach is that the time when nucleation ceases is a common time for all bubbles within a given volume of melt, regardless of their time of nucleation. Pegging this time for any bubble, permits the reconstruction of a nucleation history. Our model accounts for the joint exsolution of H<sub>2</sub>O and CO<sub>2</sub> and we find that main effect of CO<sub>2</sub> is to decrease the rates at which bubbles grow after nucleation. However, because of the relatively low CO<sub>2</sub> concentrations in Novarupta dacites, this effect is not of importance.

[72] Because we estimate bubble nucleation rates from precise diffusive bubble growth calculations, we minimize the large uncertainties associated with surface tension. We predict magma decompression rates of the order of  $10^7$  Pa s<sup>-1</sup>, consistent with conduit flow models [e.g., Koyaguchi, 2005; Massol and Koyaguchi, 2005] and approximately one order of magnitude lower than would be predicted by the bubble number density decompression rate meter of Toramaru [2006]. Our modeled vesicle size distributions for the Novarupta dacite samples are consistent with conditions that correspond to the potential energy threshold for magma fragmentation. Therefore, the highest nucleation rates

appear to coincide with magma fragmentation, supporting the notion that feedbacks between both processes are potentially self-sustaining [e.g., Massol and Koyaguchi, 2005].

[73] In 1912 Novarupta dacite that formed the pyroclasts analyzed herein, bubble growth was rapid. The smallest vesicles, which are also the most abundant, grew to their final size within a few hundred milliseconds. Interestingly, our model results suggest that only a fraction of the pre-eruptive dissolved volatiles, that is 10–20%, is exsolved from the melt at the time of magma fragmentation. The implication is that the 1912 Novarupta pyroclasts either retain a substantial amount of magmatic volatiles in their matrix glass, or that the majority of magmatic volatiles was lost by open-system degassing after magma fragmentation and before quenching.

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