

Stronger or longer: Discriminating between Hawaiian and Strombolian eruption styles

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ABSTRACT

The weakest explosive volcanic eruptions globally, Strombolian explosions and Hawaiian fountaining, are also the most common. Yet, despite over a hundred years of observations, no classifications have offered a convincing, quantitative way of demarcating these two styles. New observations show that the two styles are distinct in their eruptive time scale, with the duration of Hawaiian fountaining exceeding Strombolian explosions by ~300–10,000 s. This reflects the underlying process of whether shallow-exsolved gas remains trapped in the erupting magma or is decoupled from it. We propose here a classification scheme based on the duration of events (brief explosions versus prolonged fountains) with a cutoff at 300 s that separates transient Strombolian explosions from sustained Hawaiian fountains.

INTRODUCTION

Kīlauea, Hawaii (USA), and Stromboli, Aeolian Islands (Italy), are among the most intensely monitored, continually active volcanoes in the world, and their activity has given rise to two of the most frequently used names for eruption styles, Hawaiian and Strombolian. Both styles are also well represented in the recent eruptions at Etna, Italy. Continuity of eruptive activity and of real-time geophysical and geochemical observations makes these three volcanoes natural sites to delineate these eruption styles rigorously.

Recent debate within the volcanological community clearly emphasizes that the confusion in characterizing and classifying eruptions has greatly hindered our capability to identify potential eruptive scenarios and assess the associated hazards at these and other volcanoes (Bonadonna et al., 2014). This is particularly crucial in the case of small-scale eruptions, which are the most frequent but the most difficult to characterize, mostly due to limited dispersal of the products and/or brief durations. The characterization and classification of volcanic eruptions are crucial not only to our scientific understanding, but also for hazard and risk assessment, as well as communication to the public. Kīlauea, Etna, and Stromboli are locations of large and growing volcano-tourism operations. Their eruptions pose particular issues for management agencies because the volcanoes are highly accessible. Hawaii Volcanoes National Park records ~5000 visitors per day to the summit of Kīlauea, while the population of

Stromboli increases tenfold to ~4000 people in the summer tourist season. Etna, a UNESCO world heritage site since 2013, is one of the most visited volcanoes in the world.

CLASSIFICATIONS

Both eruption names were introduced qualitatively, based on direct observations of eruptions at these volcanoes (Mercalli, 1881; Macdonald, 1972). The two styles were sub-

sequently first classified quantitatively on the basis of deposit characteristics by Walker (1973), using principally the rate at which the products thin with distance from vent as some measure of dispersal of the ejecta, which in turn is a proxy for mass discharge rate (intensity). By these criteria, collectively all Hawaiian and Strombolian eruptions are “weak” with low mass eruption rate, as they have limited ranges of tephra dispersal and form steep-walled pyroclastic cones or ramparts rather than areally extensive sheet-like deposits. A major issue with the use of the Walker classification for weak eruptions arises because no Hawaiian deposits and no products of eruptions at Stromboli and Etna were used in arriving at this classification. In fact, contrary to the Walker classification, the data presented here show that normal Strombolian activity is weaker (in terms of mass eruption rate, i.e., kg/s), not stronger, than Hawaiian fountains (Fig. 1). Consequently, subsequent

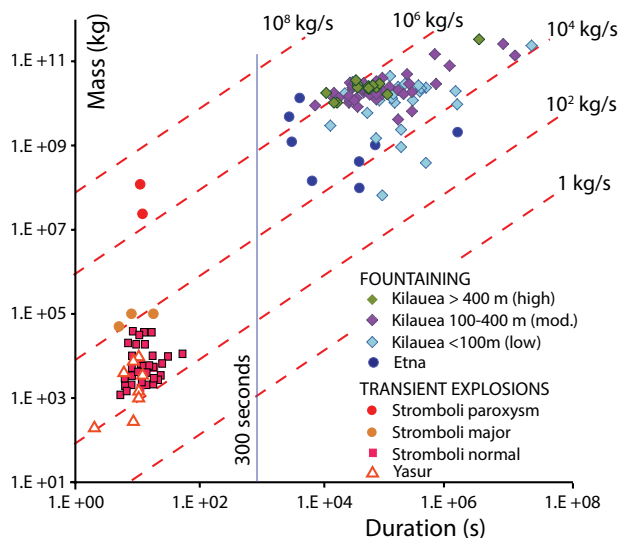


Figure 1. Plot of duration (derived either by direct observation or analysis of webcam records) versus erupted mass for selected 20th- and 21st-century explosive activity at Stromboli (Italy), Etna (Italy), and Kīlauea (Hawaii, USA). Also included are eight explosions at Yasur (Vanuatu) which appear to define the short-duration, small-mass end member amongst normal Strombolian activity. Red dashed lines connect points of equal mass discharge rate. All references for these eruptions are provided in the GSA Data Repository¹. mod.—moderate.

¹GSA Data Repository item 2016047, supplementary notes and Figure DR1 summarizing data and data sources for text Figures 1 and 4, and including an enlargement of the short-duration, small mass portion of Figure 1, is available online at www.geosociety.org/pubs/ft2016.htm, or on request from editing@geosociety.org or Documents Secretary, GSA, P.O. Box 9140, Boulder, CO 80301, USA.

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classifications avoided delineating Hawaiian and Strombolian, by either excluding Hawaiian (Pyle, 1989) or grouping Strombolian and Hawaiian together (Bonadonna and Costa, 2013).

A quantitative demarcation between the two styles, however, would be particularly useful, because eruptive activity at basaltic volcanoes shifts frequently between both eruptive styles (Spampinato et al., 2012). Three volcanoes, Stromboli, Kīlauea, and Etna, are of exceptional value to address quantitative classification of basaltic explosive eruptions, as both duration and erupted mass are known for numerous events. Elsewhere, durations of Strombolian and Hawaiian events are generally well constrained, but there is a paucity of data for erupted mass and hence mass discharge rate, due both to their local dispersal and the high risk in the near field. For this reason we explore possible classification criteria using initially well-constrained eruptions at Kīlauea, Stromboli, and Etna. We then use a larger data set of events of known duration as validation for our new approach.

EXPLOSIONS AT STROMBOLI

Stromboli, the “type locality” for Strombolian explosions, has shown an extraordinary level and diversity of activity for at least 1300 yr (Rosi et al., 2013; Taddeucci et al., 2015). Eruptions have been described qualitatively (Table 1) as normal, major, or paroxysmal explosions (Rosi et al., 2013). Normal activity (Fig. 2) typically involves <20-s-long explosions, which eject centimeter- to meter-sized pyroclasts to heights of 50–400 m (Rosi et al., 2013), on time scales of <5 to >25 events per hour. Data of Rosi et al. (2013) suggest that the durations of normal explosions range between 1.3 and 30 s (mean 7 s). In the most detailed analysis of individual events, Patrick et al. (2007) listed 136 explosions recorded in June–July 2004 with durations between 6 and 41 s (average 15 ± 6 s). The erupted mass of normal explosions has been estimated at between 1 and 10^4 kg (Ripepe et al., 1993; Harris et al., 2013; Gaudin et al., 2014; Bombrun et al., 2015). The high variability of mass ejected during each event also led to classification issues among the normal Strombolian events (Leduc et al., 2015). Recent use of high-speed imagery (Gaudin et al., 2014; Taddeucci et al., 2015) shows that each normal explosion consists of multiple sub-second pulses, each releasing a meter-diameter pocket of gas. A similar range of erupted mass and duration was also recorded during normal Strombolian explosions at Yasur volcano, Vanuatu (Fig. 1), during 10–12 July 2011 (Gaudin et al., 2014).

Larger events known as “major explosions” are recorded several times each year (e.g., Gurioli et al., 2013), while paroxysms occur “every few decades” (Rosi et al., 2013, p. 472). Both are related to the rapid rise of gas-rich magma and are characterized by durations of

TABLE 1. SUBCLASSES OF ACTIVITY AT STROMBOLI

Eruption subclass	Mass (kg)	Frequency	Volcanic Explosivity Index	Duration (s)	Repose (s)
Normal	$1\text{--}10^4$	Several per hour	–3 to –6	1–10	$10^2\text{--}10^4$
Major	$10^4\text{--}10^6$	1–8 per year	–3 to 0	~10	$10^5\text{--}10^6$
Paroxysm	$10^7\text{--}10^9$	0–4 per decade	0 to 1	$10\text{--}10^2$	$10^8\text{--}10^9$

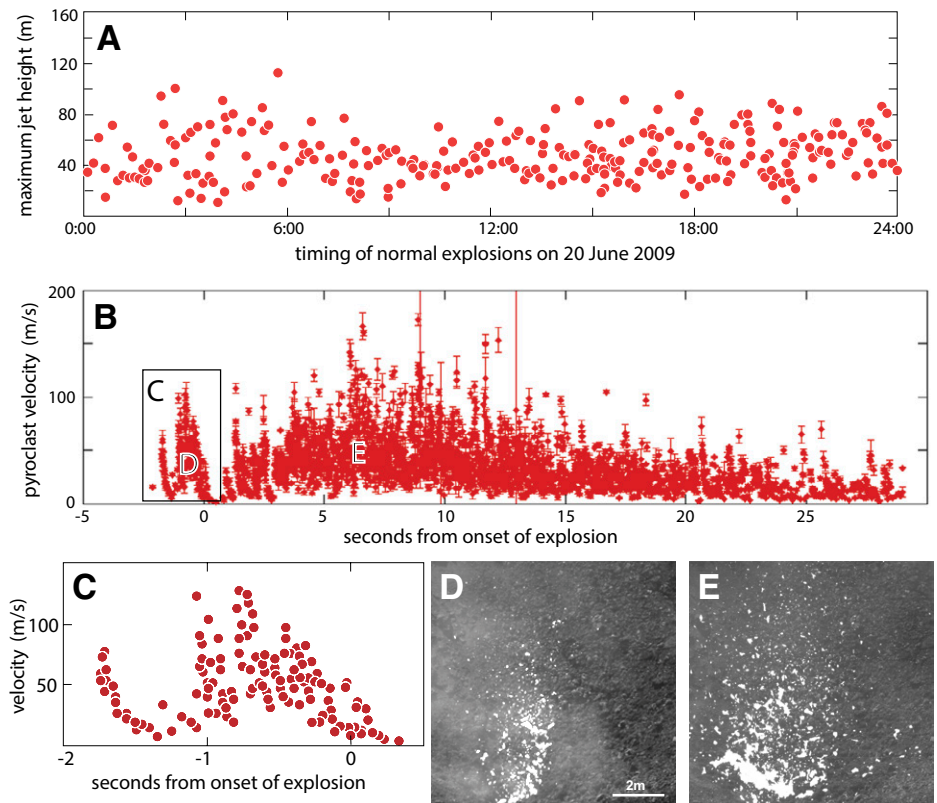


Figure 2. Examples of normal Strombolian events. A: Plot of our unpublished data (J. Taddeucci) showing discrete explosions recorded over a 1 d interval on 20 June 2009. **B:** Plot of pyroclast exit velocity used to delineate multiple pulses during a single 28-s-long explosion on 20 June 2009. **C–E:** Extension of 2 s time interval within B showing velocity measurements for individual pyroclasts during three pulses (C), and images showing initial (D) and strongest (E) pulses during event captured in B. Scale bar is the same for panels D and E. All references for these eruptions are provided in the Data Repository (see footnote 1).

tens of seconds to a few minutes and eruptive masses of $10^4\text{--}10^5$ kg and $10^7\text{--}10^9$ kg, respectively. Although mass discharge rates for paroxysms overlap with those of Hawaiian fountains (Fig. 1), all three types of activity at Stromboli are of short duration relative to Hawaiian activity. Background activity to all types of explosive eruptions at Stromboli consists of two forms of shallow-derived outgassing: passive gas streaming and small gas bursts (“puffing”) (Burton et al., 2007; Harris and Ripepe, 2007).

FOUNTAINS AT KĪLAUEA

Kīlauea, the reference volcano for Hawaiian fountaining, has been in near-continuous eruption since A.D. 1983. Forty-seven (47) Hawaiian fountaining episodes were recorded at Pu‘u Ō‘ō between January 1983 and July 1986, each

sustained at fountain heights of 30–470 m for at least 5 h and up to 12 d, erupting 4×10^9 kg to 7×10^{10} kg of magma (Wolfe et al., 1988). Single fountaining episodes during two other prolonged eruptions, in 1959 and 1969, had fountain heights of 30–579 m, were sustained between 2 h and 7 d, and erupted masses of 3×10^9 kg to 1×10^{11} kg (Richter et al., 1970; Swanson et al., 1979). These fountains are clearly distinguished from any Strombolian explosions by their longer durations (Fig. 1) despite almost total overlap in erupted mass and mass eruption rates with Strombolian paroxysms. Hawaiian fountains are sustained in the sense that continuous mass discharge is maintained for hours to days, but are also unsteady in nature, i.e., fluctuate in height and mass eruption rate at frequencies of up to 1 Hz (Fig. 3).

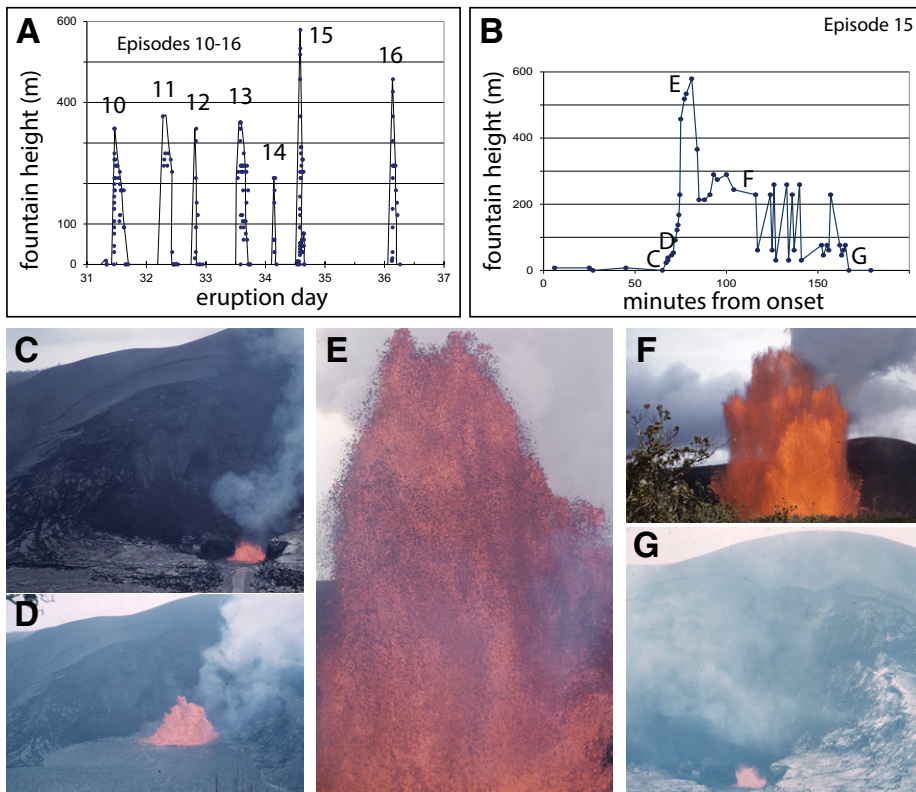


Figure 3. Examples of Hawaiian fountaining behavior. **A:** Fountain height with time for seven fountaining episodes over 5 d at close of 1959 Kīlauea Iki eruption. **B:** Enlargement of plot in **A** for episode 15, the highest fountain ever recorded at Kīlauea. **C–G:** Photos from labeled points in **B**. Like many Hawaiian episodes, fountain builds rapidly from weak onset (**C**), to low sustained fountaining (**D**), reaches short-lived maximum height (**E**), then stabilizes at lower level (**F**) before entering an unsteady phase prior to close of episode (**G**). Data after Richter et al. (1970).

EXPLOSIVE ERUPTIONS AT ETNA

Etna has had an extraordinary frequency, and diversity, of Strombolian to subplinian activity since 1990. Etna is an invaluable third “type” volcano because, while Kīlauea is dominantly Hawaiian in style and Stromboli is overwhelmingly Strombolian, Etna’s explosivity offers a third perspective as activity is episodic: while some explosive episodes are purely Strombolian, others are purely fountaining and some show alternations of both styles, often on time scales of hours or less. Transitions between normal Strombolian explosions and fountaining have occurred repeatedly in the 21st century (Andronico et al., 2005, 2014). Transitions are rapid and marked by a short period of increased frequency of Strombolian explosions (“rapid Strombolian”) before the sharp onset of sustained fountaining. The tempo of eruption at Etna has increased steeply since 1998, with numerous fountaining episodes now recorded every year (Andronico et al., 2014).

A NEW APPROACH TO CLASSIFICATION

A large gap exists, from 10² to 10⁴ sec, between the typical duration of transient explo-

sions and that of fountains at Kīlauea, Etna, and Stromboli. In comparison, overlaps in terms of both erupted mass and mass discharge rate rule out either of these parameters as a principal basis to distinguish these two eruptive styles (Fig. 1). Based on the typical durations of events in Figure 1, we propose a classification for low-intensity explosive eruptions in which the first-order criterion is duration of the event. We suggest that a natural division between Strombolian explosions of all sizes and Hawaiian fountaining episodes is a duration of 300 s, close to the middle of this wide gap.

We can test the validity of using duration as a parameter to separate Hawaiian and Strombolian eruptions by looking at an extended data set that includes activity where event durations are well constrained but no estimates exist for eruptive mass. This includes a much larger number of fountaining episodes at Etna in 2000 and 2011, plus transient Strombolian explosions at Yasur, Mount Erebus (Antarctica), and Villarrica (Chile) volcanoes (Fig. 4). Across all of these data, for 860 events, there is a gap between 40 s and 1.2 × 10³ sec with no recorded events.

For Strombolian eruptions, there are at this time insufficient data for larger eruptions to ex-

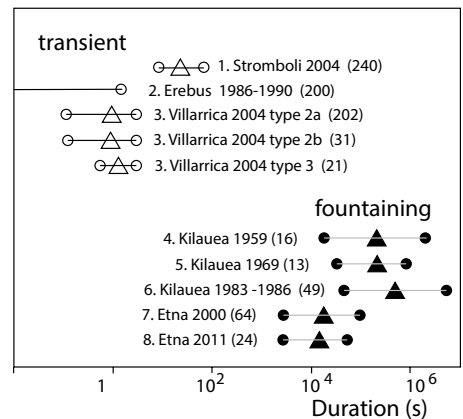


Figure 4. Plot of event durations for well-constrained sequences of transient Strombolian explosions (open symbols) and sustained Hawaiian fountaining (filled symbols) at Stromboli (Italy), Mount Erebus (Antarctica), Villarrica (Chile), Kīlauea (Hawaii, USA), and Etna (Italy) volcanoes. Number of sampled events is indicated in parentheses, following year(s) of events. Triangles are average durations in seconds, circles represent longest and shortest events. Mount Erebus is a special case in which every explosion lasted <1 s, and represented bursting of a single short-lived bubble. Villarrica explosions were divided by Gurioli et al. (2008) into three groups. Type 1 events comprised gas-only emissions; type 2, involving emission of gas and ejecta, were divided into 2a and 2b, which involved less heavily and more heavily loaded ejecta clouds respectively; type 3 events involved ejection of coherent sheets of magma and detached blebs. All references for these eruptions are provided in the Data Repository (see footnote 1).

tend the threefold classification used at Stromboli for use elsewhere. However we propose the addition of a category called *rapid* explosions to represent sequences of very closely spaced and, generally, very weak explosions, with a periodicity at least two orders of magnitude higher than that of normal explosions at Stromboli. Such activity has been seen and recorded on surveillance cameras at Stromboli, Etna, and Yasur (Andronico et al., 2005; Gaudin et al., 2014).

For Hawaiian fountains, any informal sub-classification based on erupted mass is less meaningful, as some eruptions occur from long fissures and others from point sources, and some eruptions are of low mass eruption rate but long duration and vice versa. Both low and very high fountains can thus have comparable erupted mass, depending on the surface area of the vent and the duration of the eruption. For example, the 1959 Kīlauea Iki episode 16 from a point vent erupted 10¹⁰ kg of magma in 3 h, with a peak height of 457 m (Richter et al., 1970). Episode 1 of the Mauna Ulu 1969 eruption ejected a comparable mass over 34 h from a 4-km-long fissure (Swanson et al., 1979) with a peak height of <50 m. Instead, we propose an informal split into low, moderate, and high fountaining at sus-

tained fountain heights of <100 m, 100–400 m, and >400 m (Table 2).

TABLE 2. PROPOSED SUBCLASSES OF HAWAIIAN FOUNTAINING

Hawaiian class	Peak height (m)
High	>400
Moderate	100–400
Low	<100

“MISFITS”: OTHER ERUPTION STYLES AT KĪLAUEA AND STROMBOLI

Other styles of magmatic activity occur at both volcanoes. These include passive outgassing and puffing, weak spattering, gas pistoning, and non-explosive effusion of lava. A comprehensive classification will need to include these but is beyond the scope of this paper, which merely addresses the more tractable part of the classification problem.

CONCLUSIONS

Distinction between Strombolian and Hawaiian eruptions is part of a more generic issue in that existing deposit-focused quantitative classifications cannot distinguish between sustained and transient eruption styles, i.e., between Hawaiian, subplinian, and Plinian eruptions versus Strombolian and Vulcanian explosions. This is arguably a first-order distinction in physical volcanology, linked to the extent to which shallow exsolved gas remains mechanically coupled to, or decoupled from, the melt phase in the very shallow conduit. The problem exists not only for Hawaiian and Strombolian eruptions, but also at higher mass eruption rates where subplinian and Vulcanian eruptions also cannot be distinguished on deposit characteristics alone. To be functional, any unambiguous classification of these eruptive styles also requires inclusion of some measure of event duration. More data are perhaps needed to address the subplinian versus Vulcanian issue and the separation between Vulcanian and Strombolian activity, and we hope this paper will provoke that debate.

An unresolved issue is what criteria can be applied to classify unobserved prehistorical eruptions and products as Strombolian or Hawaiian. The outlined classification neither improves nor worsens the situation, as no other system has ever worked for these events either. A textural criterion, based on the fact that Strombolian eruptions typically involve slightly more-viscous magmas and produce more ragged pyroclasts whereas Hawaiian deposits are rich in fluidal achenliths reflecting lower viscosity, is a possibility if such a contrast can be borne out by the componentry of eruptions at Kīlauea, Etna, and Stromboli (Taddeucci et al., 2015).

ACKNOWLEDGMENTS

The authors wish to acknowledge grants from the National Science Foundation (grants EAR-0409303, EAR-0810332, EAR-1145159, EAR-1427357) and the American Recovery and Reinvestment Act (grant 113153 via the Hawaiian Volcano Observatory), which funded this research. We are also grateful to Jim Kaahikaua for his support throughout the study and to Maria Janebo and Samantha Weaver for review of the manuscript and invaluable assistance in the field. We highly appreciate insightful constructive reviews by Kimberly Genearau and especially Lucia Gurioli, Letizia Spampinato, Heather Wright, and an anonymous reviewer.

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Manuscript received 12 October 2015
 Revised manuscript received 18 December 2015
 Manuscript accepted 23 December 2015

Printed in USA